FINITE SAMPLE PROPERTIES OF TESTS OF THE EPSTEIN-ZIN ASSET PRICING MODEL

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Abstract

This paper investigates the small sample properties of Hansen and Singleton (1982)-type GMM tests of asset pricing restrictions implied by Epstein and Zin (1989) preferences. The Monte Carlo results suggest that tests of the Epstein and Zin (1989) asset pricing model often have little size-adjusted power to reject asset pricing restrictions implied by simpler, time and state separable expected utility preferences, even when parameters are chosen to make the difference between the relative risk aversion parameter and the reciprocal of the intertemporal substitution parameter large. There is evidence that a Wald test has greater power than other tests and that use of Hansen, Heaton and Yaron's (1996) continuous-updating GMM estimator improves the power of the tests.

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1. Introduction

The consumption-based asset pricing model of Rubinstein (1976), Lucas (1978) and Breeden (1979) provides an important theoretical description of the cross-sectional and intertemporal behavior of asset returns. Despite its intuitive appeal, econometric tests consistently reject the model (e.g. Grossman and Shiller, 1981; Hansen and Singleton, 1982; Mehra and Prescott, 1985; Cochrane, 1992). The model's poor performance in empirical tests implies a violation of the assumptions underlying the model. For example, the expected utility preferences used to derive the consumption-based model typically restrict the representative agent's risk aversion parameter to equal the reciprocal of the agent's elasticity of intertemporal substitution parameter (Hall, 1988). This assumed inseparability of the desire to hedge risk from the desire to smooth consumption may itself lead to rejections of the consumption-based model in empirical tests. Epstein and Zin (1989) develop a class of non-expected utility preferences that nests time and state separable expected utility preferences (TSEU) as a special case, but separates the agent's relative risk aversion and elasticity of intertemporal substitution parameters.[1]

I study the small sample properties of GMM estimators and tests based on an asset pricing model implied by Epstein-Zin (EZ) preferences. Specifically, I investigate the one-period-forward Euler equation tests popularized by Hansen and Singleton (1982) and studied in Epstein and Zin (1991). The small sample results are derived from Monte Carlo simulations using an endowment process calibrated to mimic the joint dependencies observed in aggregate consumption and dividend data in the U.S. In this manner, my investigation extends to EZ preferences earlier small sample GMM studies of the TSEU model (Tauchen, 1986; Kocherlakota, 1990b; Hansen, Heaton and Yaron,1996).

A more novel feature of my paper is its focus on the *power* of GMM hypothesis tests. I investigate the power of a variety of tests to reject the TSEU null in favor of the EZ alternative, using parameters chosen to match estimates from Epstein and Zin (1991), as well as “extreme” parameters that imply risk preferences that deviate greatly from TSEU preferences. I also use the extreme parameterizations to investigate the sensitivity of my results across different sample sizes, instrument sets, estimation methods, and parameterizations of the data generating process.

I find that when the model parameters are chosen to be close to the estimates in Epstein and Zin (1991), hypothesis tests formed from the single-period EZ Euler equation restrictions have no size-adjusted power to reject the TSEU null. In fact, the proportion of rejections rarely exceeds the small sample size of the test. When population parameters are chosen to make the difference between the risk aversion and intertemporal substitution parameters extreme, the power of the Wald test improves somewhat, while the power of the Newey and West (1987) D-test and Hansen’s (1982) goodness-of-fit test remains low. The low power results are robust across to changes in the sample size, specification of the endowment process, choice of instruments and methods for estimating the GMM weighting matrix. I do find some evidence that the continuous-updating estimator, introduced by Hansen, Heaton and Yaron (1996), improves the power of the D and goodness-of-fit tests.

I conjecture that the power of the tests is low because the GMM variance-covariance matrix suffers from a type of collinearity that greatly attenuates the preciseness of the parameter estimates. To support this conjecture, I report diagnostics suggested by Belsley (1991). While collinearity in a linear world is typically a data problem, I argue that the functional form of the nonlinear Euler equation studied here is predisposed to create high linear dependencies among the columns of the parameter covariance matrix. Since a variety of popular asset pricing models are estimated and tested using a similar functional form to the one studied here, low power may be a concern in other tests of nonlinear asset pricing models.

The plan for the remainder of the paper is as follows. Section 2 briefly summarizes the EZ asset pricing model and reviews GMM methodology. Section 3 describes the data generating process, the Monte Carlo simulation methodology and provides some initial background summary statistics. Section 4 reports the bulk of the results and Section 5 concludes the paper.

2. Estimating Epstein-Zin Using GMM

2.1 Epstein-Zin Preferences

With Epstein-Zin preferences, an infinitely-lived representative agent makes optimal consumption and portfolio choices based on the following recursive function:

$$U_t = \left[(1-\beta)c_t^{1-\rho} + \beta(E_t U_{t+1}^{1-\alpha})^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}}, \qquad (1)$$

where $c_t > 0$ is the agent's consumption at time t, E_t is the conditional expectation taken with respect to information available at time t, $\alpha > 0$ is the agent's relative risk aversion (RRA) parameter, $\beta < 1$ is the subjective discount factor and $\sigma = 1/\rho$ ($\rho > 0$) is the elasticity of intertemporal substitution (EIS) parameter. The function $(E_t U_{t+1}^{1-\alpha})^{1/1-\alpha}$ is the certainty equivalent of next period utility. An agent with risk aversion parameter $\alpha^* > \alpha$ is said to be more risk averse when $(E_t U_{t+1}^{1-\alpha^*})^{1/1-\alpha^*} \leq (E_t U_{t+1}^{1-\alpha})^{1/1-\alpha}$. Holding the certainty equivalent of next period's utility constant, equation (1) becomes a constant elasticity of substitution function, where the parameter ρ governs the agent's sensitivity to tradeoffs between current consumption and the certainty equivalent of future utility.[2]

Let $R_{j,t+1}$ be the random, real return on asset j at time t+1 and let the subscript w denote the claim on the portfolio of aggregate wealth. Epstein and Zin (1991) show that optimizing (1) subject to standard budget constraints, yields the following set of Euler equations,

$$E_t\left[\tilde{m}_{t+1}\tilde{R}_{j,t+1}\right]=1, j=1,2,\ldots,N,$$

$$\text{where } \tilde{m}_{t+1}=\left(\beta\left(\frac{\tilde{c}_{t+1}}{c_t}\right)^{-\rho}\right)^{\frac{1-\alpha}{1-\rho}}\tilde{R}_{w,t+1}^{\frac{1-\alpha}{1-\rho}-1}. \qquad (2)$$

Since the marginal rate of substitution m_{t+1} depends on both the consumption growth rate *and* the return on aggregate wealth, the EZ Euler equation inherits characteristics from both the consumption-based asset pricing models (Rubinstein, 1976; Lucas, 1978; Breeden, 1979) and the standard single-period capital asset pricing model (Sharpe, 1964; Lintner, 1965). [3]

2.2 Epstein-Zin Preferences and Asset Returns

There are characteristics of the EZ specification that are potentially appealing to researchers wishing to describe the behavior of asset returns. For instance, the risk aversion parameter determines how an agent divides current investable wealth across the assets in a portfolio at particular points in time. The substitution parameter, on the other hand, governs the choice between how much to consume today versus the future, which in turn, dictates the dollar amount of wealth to be invested.

Evidence in Epstein and Zin (1991), Kandel and Stambaugh (1991), Ferson and Constantinides (1991), Campbell (1993) and Hardouvelis, Kim and Wizman (1996) suggests that the risk aversion parameter may influence the cross-sectional characteristics of expected returns, while the substitution parameter may influence more the time-series behavior of expected returns. Moreover, Kandel and Stambaugh (1991) and Campbell (1993) argue that EZ asset pricing models help jointly explain anomalies such as the equity premium puzzle, high return volatility and the long-horizon serial dependence in equity returns.

Weil (1989) and Kocherlakota (1990a) note that when consumption growth is independent and identically distributed (i.i.d.) through time, the testable restrictions implied by the EZ model are observationally equivalent to the TSEU model. Intuitively, the i.i.d.

assumption transforms the agent's choice to a single-period problem, rendering the intertemporal substitution parameter useless. Kocherlakota (1990a) conjectures that the power of tests to reject a TSEU null in favor of the EZ model will depend on the degree of serial dependency in consumption growth.

2.3 Estimation and Testing Using GMM

The generalized method of moments (GMM) estimation strategy developed in Hansen (1982) exploits orthogonality conditions such as (2), while requiring only relatively weak assumptions on the distribution from which the sample data is drawn. Consider imposing orthogonality conditions of the form,

$$E[g(x_{t+1}, z_t, b_0)] = 0,$$

$$g(x_{t+1}, z_t, b_0) = u(x_{t+1}, b_0) \otimes z_t, \tag{3}$$

where, E is the unconditional expectations operator, the elements of the *k*-vector b_0 are unobservable parameters to be estimated, x_{t+1} is a $l \times 1$ vector of variables, $u(x_{t+1},b_0)$ is a function of dimension *d* and z_t is a $r \times 1$ vector of instruments observable to the econometrician at time t. The *dr*-vector of orthogonality conditions $g(x_{t+1},z_t,b_0)$ is obtained by multiplying the vector, $u(x_{t+1}, b_0)$ by each element in the vector z_t. Using the restriction in Equation (2), the *j*th element of $u(x_{t+1},b_0)$ in the context of this paper is

$$u_j(x_{t+1}, b_0) = \beta^{\gamma}\left(\frac{c_{t+1}}{c_t}\right)^{-\rho\gamma} R_{w,t+1}{}^{\gamma-1} R_{j,t+1} - 1, \tag{4}$$

where $\gamma \equiv \frac{1-\alpha}{1-\rho}$, $x_{t+1} = \left(\frac{c_{t+1}}{c_t}, R_{w,t+1}, R_{1,t+1}, \ldots, R_{N,t+1}\right)$, $b_0 = (\beta, \gamma, \rho)$, c_{t+1}/c_t is a measure of the growth in real per-capita consumption, $R_{w,t+1}$ is a proxy for the gross, real return on the aggregate wealth portfolio and $R_{j,t+1}$, $j = 1, 2, \ldots N$, is the gross, real return on the *j*th asset. Since Equation (2) implies $E_t[u(x_{t+1},b_0)] = 0$, it follows by iterative expectations that, $E[u(x_{t+1},b_0)\otimes z_t] = E[g(x_{t+1},z_t,b_0)] = 0$.

The method of moments estimator replaces $E[g(x_{t+1},z_t,b_0)]$ with its sample analog,

$$g_T(b) = \frac{1}{T}\sum_{t=1}^{T} g_t(x_{t+1}, z_t, b) \quad (5)$$

that for large T, should be close to zero when evaluated at b_0. Estimating b_0 can be accomplished by choosing b_T to minimize the quadratic,

$$Q_T(b) = g_T(b)' W_T g_T(b)\,, \quad (6)$$

where W_T is a $dr \times dr$ symmetric, positive semi-definite weighting matrix.

The asymptotic covariance matrix of the GMM estimator b_T depends on the choice of the weighting matrix W_T. Hansen (1982) shows that fixing z_t, the choice of W_T that leads to the smallest asymptotic covariance matrix of b_T, within the class of GMM estimators, is $W_T = V_T^{-1}$, where V_T is a consistent estimate of the asymptotic covariance matrix of $T^{1/2}g_T(b_0)$. Conditional on several assumptions[4], Hansen (1982) also shows that b_T is a consistent estimator of b_0 and is asymptotically normally distributed with covariance matrix, $(G_0' V^{-1} G_0)^{-1}$, where $G_0 = E[\partial g_T(b_0)/\partial b_0]$ and $V = E[g_t(b_0)g_t(b_0)']$. When the system is over-identified ($dr > k$), there remain *dr-k* "free" restrictions, not used to estimate b_0, that can be used to test the goodness-of-fit of the model. If the model is correct, the value $J_T = T \bullet Q_T(b_T)$ is asymptotically chi-squared, with *dr-k* degrees of freedom. Thus the statistic J_T provides a measure of the goodness-of-fit of the model.

2.4 Hypothesis Tests

Under the null hypothesis of TSEU preferences, the ratio $\gamma \equiv \frac{1-\alpha}{1-\rho}$ is equal to one. The power of tests to reject TSEU will essentially depend on the ability to statistically distinguish estimates of

γ from one. I investigate the power to reject the TSEU null using three different test statistics. The first, termed a "D-statistic" by Newey and West (1987), is analogous to a Likelihood Ratio Test. It fixes the value of the weighting function W_T and then measures the normalized distance between the minimized restricted and unrestricted objective functions. Using the weighting matrix estimated under the alternative assures that the resulting test statistic has a well-specified asymptotic distribution under both the null and alternative. The second is a Wald test, or essentially a squared t-statistic, based on the estimate of γ centered around one. The third statistic is the value of the goodness-of-fit test under the TSEU null, $J_{T,\,TSEU}$.

3. Background for Experiments

3.1 Monte Carlo Setup

To conduct the Monte Carlo simulations, I first draw T observations on consumption growth and asset returns from a data generating process (DGP) constructed to be jointly consistent with features of actual consumption and dividend data and a representative EZ agent. I then obtain estimates of the preference parameters and asymptotic standard errors and construct test statistics from a given GMM estimator. This procedure, when repeated 500 times, generates empirical distributions of model parameter estimates, standard error estimates and the test statistics.

3.2 The Data Generating Process

Following Tauchen (1986), Kocherlakota (1990b) and Hansen, Heaton and Yaron (1996), I adopt Tauchen and Hussey's (1991) method for approximating solutions to integral equations using finite-state Markov chains to construct a DGP. Specification of the DGP begins with the process for two endowment variables, real aggregate dividend growth and real per-capita consumption growth. The Tauchen and Hussey (1991) procedure first calibrates a set of conditional probability weights and state values of a Markov process to mimic a continuous vector autoregression (VAR) on the endowment variables. Given the finite-state endowment process, state prices for the EZ securities can then be computed by fixing the preference parameters and solving a discrete version of equation (2). The details of this computation are in the appendix.

For the bulk of the reported results, the Tauchen and Hussey (1991) approximation is used to match the following VAR(2) estimate of the logarithm of *annual* per-capita, real non-durable consumption growth ($\lambda_t = c_t/c_{t-1}$) and the logarithm of annual S&P 500 aggregate, real dividend growth ($\zeta_t = d_t/d_{t-1}$) over the period 1888-1978,

$$\begin{aligned} \ln \zeta_t &= 0.004 + 0.117 \ln \zeta_{t-1} + 0.414 \ln \lambda_{t-1} + \varepsilon_t^1 \\ \ln \lambda_t &= 0.021 + 0.017 \ln \zeta_{t-1} - 0.161 \ln \lambda_{t-1} + \varepsilon_t^2 \end{aligned} \tag{7}$$

$$\text{var}(\varepsilon) = \begin{bmatrix} 0.01400 & 0.00177 \\ 0.00177 & 0.00120 \end{bmatrix}.$$

The parameters from the above model are from Kocherlakota (1990b), who uses data from Mehra and Prescott (1985). The Markov approximation assumes the errors $\boldsymbol{\varepsilon} = (\varepsilon^1_t, \varepsilon^2_t)'$ are jointly normally distributed with covariance matrix given by **var(ε)**. For robustness purposes, I also include a parameterization of the VAR(2) from Hansen, Heaton and Yaron (1996),

$$\begin{aligned} \ln \zeta_t &= 0.0012 - 0.1768 \ln \zeta_{t-1} + 0.1941 \ln \lambda_{t-1} + \varepsilon_t^1 \\ \ln \lambda_t &= 0.0019 + 0.0267 \ln \zeta_{t-1} - 0.2150 \ln \lambda_{t-1} + \varepsilon_t^2 \end{aligned} \tag{8}$$

$$\text{var}(\varepsilon) = \begin{bmatrix} 0.1438 & 0.0001 \\ 0.0001 & 0.0145 \end{bmatrix} \times 10^{-3},$$

They estimate the VAR using *monthly* aggregate data from the period 1959-1992, using U.S. expenditures on non-durables and services for per-capita consumption and monthly dividends (seasonally adjusted) from the value-weighted CRSP portfolio. Note that the estimated consumption processes in both equations (7) and (8) exhibit negative serial correlation, while current dividend growth is positively related to lagged dividend growth in the annual data, but negatively related in the monthly model.

To complete the specification of the DGP, I use the parameters from the assumed endowment process, along with values for the subjective discount factor β, relative risk aversion α, and elasticity of intertemporal substitution ρ parameters to generate state returns for three simulated assets.[5] The first return R_w is from the claim on aggregate wealth. For the bulk of the simulations, R_w is the return on a claim whose dividend equals per-capita consumption. That is, I assume the representative agent receives only dividends as income and consumes all dividends when received. I explore variations on this assumption in Section 4.5. The second return R_{sp} is from a claim on the S&P 500 dividend stream (for monthly data, the CRSP dividend stream). The third return R_f is the risk-free rate; it is defined to be the return on a claim paying a unit of consumption for sure one period forward.

I abstract from measurement problems such as time-aggregation in consumption data or the unobservability of the true return on wealth by assuming that the econometrician observes the data without error. In this sense, the data generated here are unrealistically ideal. For all reported simulations, I fix the value of the subjective discount factor β to 0.98. The conclusions of the paper are not changed by using other values for β.

The first set of parameterizations is based roughly on the maximum and minumun estimates of the RRA parameter α and intertemporal substitution parameter ρ obtained by

Epstein and Zin (1991).[6] For this set, I allow the true value of α to be either 0.80 or 1.35 and the true values of ρ to range over 0.80, 1.35 and 5.20. This range of parameter values is on the order of what is often considered to be economically reasonable by, for example, Friend and Blume (1975) or Hall (1988). The second set of parameters are "extreme" values of the risk aversion and substitution parameters and are included primarily to maximize the difference between α and ρ. Kandel and Stambaugh (1991) argue that a value of $\alpha = 29$ is required to solve the equity premium puzzle, while $\alpha = \rho = 13.7$ is used (in conjunction with a value of $\beta > 1$) by Kocherlakota (1990b) to solve the equity premium puzzle. Friend and Blume (1975) estimate α to be no greater than 2.0. I explore various combinations of 2.0, 13.7 and 29.0 for α and ρ when reporting the extreme results.

The high values for relative risk aversion in the extreme set of parameters not only allows for a greater difference in the values of α and ρ, but may also overcome estimation problems that potentially plague the lower valued α and ρ's. The GMM algorithm searches over values of β, ρ and γ to force the sample average

$$\frac{1}{T}\sum_{t=1}^{T}\left[\beta^{\gamma}\left(\frac{c_{t+1}}{c_t}\right)^{-\rho\gamma} R_{w,t+1}{}^{\gamma-1} R_{j,t+1} - 1\right] \otimes z_t \qquad (9)$$

as close to zero as possible. As the estimate of either γ or ρ approaches zero in the expression $\left(\frac{c_{t+1}}{c_t}\right)^{-\rho\gamma}$, large changes in the other parameter will have little impact on the sample average in (9). Thus, large fluctuations in the estimates can occur when the algorithm estimates the value of ρ or γ to be too close to zero. I discuss identification problems in more detail in section 4.6.

3.4 Summary Statistics

To gauge the extent to which the “true” parameters of the endowment process are recovered in VAR regressions using the artificial data, Panel A of Table 1 contains averages, across 500 draws of 90 observations, of the VAR coefficient estimates, standard errors and the residual covariance matrix, as well as the standard deviation of the coefficient estimates across the 500 simulations. The mean VAR coefficient and residual covariances are close to their population values. Moreover, the estimated standard errors accurately reflect the dispersion in the VAR estimates. With 90 observations, the VAR regressions adequately summarize the underlying distribution of the endowment variables.

Panel B of Table 1 reports the average mean and standard deviation of the asset returns and the mean equity premium (R_{sp}/R_f), using as population values, combinations of the maximum and minimum estimates of α and ρ obtained from Epstein and Zin (1991). Note that the mean return on each of the securities can be increased by increasing either α or ρ. However, the equity premium remains below one percent across all parameter choices. This observation is consistent with Weil (1989) and Kocherlakota (1990a), who argue that separating the risk aversion and substitution parameters does not, at these parameter values, solve the equity premium puzzle.

In Panel C of Table 1, the mean return and equity premium is reported using the extreme values of α and ρ. The last line of Panel C also reports the actual estimates from Mehra and Prescott (1985). The combination $\alpha = 29.0$, $\rho = 2.0$ provides a reasonable match of the equity premium and the first two sample central moments of R_{sp} and R_f. Note also that $\alpha = 29.0$ produces a large equity premium for all choices of ρ, while the equity premium remains low for the $\alpha = 2.0$, $\rho = 29.0$ case.

3.5 Choice of Instruments

Ferson and Foerster (1991) and Kocherlakota (1990b) suggest that the small sample behavior of GMM estimators worsen as the number of instruments becomes large. However, too few instruments reduces the asymptotic efficiency of the GMM estimator and worse, may yield tests with inadequate power. I first consider three different instrument sets containing two, three and four lagged variables. Instrument Series 1 contains a constant and the return on the S&P 500, lagged one period (1, $R_{sp,t-1}$). Series 2 adds lagged consumption growth to the Series 1 variables (1, $R_{,sp,t-1}$, λ_{t-1}). Series 3 contains the variables in Series 2 plus the time *t* risk-free rate (1, $R_{,sp,t-1}$, λ_{t-1}, $R_{,f,t}$).

Table 2 provides insight into the behavior of the instruments, $R_{sp,t-1}$, λ_{t-1} and $R_{,f,t}$, using data drawn from the annual DGP and three combinations of α and ρ: (α,ρ) = (0.80, 0.80), (29.0, 29.0), (29.0, 2.0). Panel A reports the mean slope estimates and t-ratios from instrument autoregressions using five lags, while Panel B provides similar summaries from regressions of consumption growth and the risky returns on the four instruments in Series 3.

The first thing to note from Panel A of Table 2 is that all of the instruments exhibit negative serial correlation that dies out quickly after the first lag. For consumption growth, the magnitude of the mean first-order estimate is of the same magnitude as that observed in annual (Mehra and Prescott, 1985) and monthly (Breeden, Gibbons and Litzenberger, 1989) data. However at 90 observations, the mean t-statistic does not reject the hypothesis that the first-order correlation is zero.

The risk-free return is significantly negatively autocorrelated, with mean point estimates of roughly -0.20 and mean t-values that are greater than two standard deviations from zero across all three parameterizations of the EZ parameterizations. The negative autocorrelation in the modeled risk-free return differs from real-world data, where the autocorrelation tends to be positive. The time series properties of the modeled risk-free rate appear not to be influenced by the absolute or relative magnitude of the RRA and EIS parameters.

By contrast, the properties of the S&P 500 return change substantially as the relative values of the RRA and EIS parameters are varied. When the parameters are restricted to the TSEU case ($\alpha = \rho$), the mean first order correlation estimates and associated t-statistics for R_{sp} are small and similar, independent of the magnitude of α and ρ. However, when α is fixed at 29.0 and ρ is decreased to 2.0, R_{sp} exhibits substantial first order negative correlation.[7]

Turning to Panel B, the instruments explain relatively little of the linear variation in consumption growth (roughly 5 percent) across all three EZ parameterizations and explain little of the linear variation (less than 10 percent) in the two risky returns when $\alpha = \rho$. However, when compared to predictive regressions using actual data, the adjusted R-squareds are similar in magnitude to those reported in the asset pricing literature.[8] For the $\alpha = 29.0$, $\rho = 2.0$ case, the instruments explain on average 41 and 55 percent of the variation in R_{sp} and R_e, respectively.

It is unclear what link the behavior documented in Table 2 will have to the performance of the GMM estimators and the power of tests of the EZ model. Nelson and Startz (1990a,b) document that substantial small sample biases can be introduced into linear estimators when the instruments used are "poor", or not highly correlated with the explanatory variables. Mankiw and Shapiro (1985) argue that orthogonality tests that use highly autocorrelated instruments will lead to overrejection. Their observation is related to the downward biases observed in estimates of time-series processes when the series is near a unit root. Taking Nelson and Startz (1990a,b) and Mankiw and Shapiro (1985) together, those instruments that are both poor and highly autocorrelated will fare the worst.

In an attempt to address the issue of instrument quality, I also examine the small sample properties of tests of the EZ model using instruments that are "optimal" in the sense they minimize, over the choice of all feasible instruments, the asymptotic covariance matrix of the GMM estimator (Hansen, 1985). These instruments can be thought of a set of indicator variables that span entirely the information set at time t and thus yield on average the *conditional* expectation $E_t[u_j(x_{t+1},b_0)]$, for large T. In this sense, the optimal instruments are also "good" according to the definition of Nelson and Startz (1990a,b). A drawback to the optimal instrument set is that it requires substantial knowledge about the true data generating process and is therefore impossible to construct in practice. The appendix destails the construction of the optimal instrument set.

3.6 Estimation Procedure

Minimization of the quadratic function Q_T is conducted using the Hansen-Heaton-Ogaki Gauss GMM program. For most of the simulations, the GMM estimates of the parameters and weighting matrix are obtained in two steps. Initial estimates are obtained by starting the algorithm at the true values of the preference parameters and weighting the sample moment vector of orthogonality conditions with a $dr \times dr$ identity matrix.[9] The weighting matrix W_T is then calculated from the first-round parameter estimates and used to obtain second-pass estimates of the parameters.

In addition to the two-step estimator, I also explore the small sample properties of two alternative GMM estimation methods that are asymptotically equivalent to the two step estimator. The first, suggested by Ferson and Foerster (1994), is an iterative estimator that repeatedly re-estimates the models parameters and updates the weighting matrix until the weighting matrix meets some convergence criterion.[10] Ferson and Foerster (1994) find that the small sample size properties are improved with the iterative estimator. The second, examined by Hansen, Heaton and Yaron (1996), estimates the weighting matrix simultaneously with the model parameters. An advantage of the continuous-updating estimator is that it is invariant to scale adjustments or parameter transformations in the moment conditions. Hansen, Heaton and Yaron (1996) find that the continuously-updated estimator has superior small sample size properties for a variety of test statistics based on the TSEU model.

For each realization of data, I estimate the EZ parameters β, ρ and the ratio $\gamma \left(\equiv \frac{1-\alpha}{1-\rho} \right)$ and record the value of the parameter estimates, their estimated asymptotic standard errors and the minimized value of the unrestricted objective function, which I label $J_{T,EZ}$.[11] Next, the TSEU restriction (γ = 1) is imposed and the quadratic is minimized in one stage using the weighting matrix formed from the EZ estimates. This yields the second component used in calculating the D-statistic. Finally, the TSEU restriction is estimated one more time using the two-step procedure described above and the resulting minimized objective function, labeled $J_{T,TSEU}$, is recorded.

4. Simulation Results

The results in Tables 3 through 6 derive from 500 draws of 90 annual observations. Tables 3 through 5 report results using the values of α and ρ similar to those found in Epstein and Zin (1991), while Table 6 contains results for the extreme parameter values. Table 7 focuses on the robustness of the results to varying various inputs to the Monte Carlo simulations such as sample size and choice of DGP, weighting matrix and instrument set. Table 8 contains collinearity diagnostics.

4.1 Estimates of Preference Parameters and Standard Errors

Table 3 reports the mean and median point and standard error estimates of β, ρ and γ and with the standard deviation of the point estimates across the 500 simulations for the parametric combinations (α,ρ) = (0.80, 0.80), (0.80, 5.20), (1.35, 1.35) and (1.35, 5.20). I also report summary statistics for the value of α, which I back out in each simulation from the estimates of γ and ρ.[12] I report both mean and median estimates as measures of central tendency since, as noted by Nelson and Startz (1990a) and Tauchen (1986), first and second moments of the GMM estimators may not exist for some parameterizations of the DGP. I define the bias in an estimate to be the distance between the true parameter value and the measure of central tendency.

Several interesting patterns emerge from Table 3. First, substantial biases exist in both the point and standard error estimates. Overall, *median* point estimates are closer to the true preference parameter values, while the *mean* standard error better reflects the dispersion in the estimates. Second, the magnitude of the bias in the mean and median estimates of γ is large, even when the true value of γ is one. For example, In Panel A, the estimate of γ is 15 to 21 times larger than its true value. Based on the point estimates of γ alone, one could wrongly suspect that there is a large difference between the risk aversion and intertemporal substitution parameters when the TSEU model is actually the correct model. Third, the median estimates of the asymptotic standard errors are always well below the standard deviation of the estimates across the simulations. Increasing the number of instruments (moving from Series 1 to Series 3) tends to concentrate both the median and mean estimated standard errors around biased values at a rate that exceeds any reduction in the actual dispersion of the point estimates.

Ferson and Foerster (1994) propose an adjustment factor to correct for the downward bias in GMM standard error estimates. In their study of the small sample properties of latent variable asset pricing models, they find that multiplying the asymptotic variance estimate by $\frac{(d+r)T}{(d+r)T-Q}$, where $Q = 0.5(kr + (d - k) + (dr)^2 + dr)$, reduces the bias in standard error estimates. For Series 1 here, such an adjustment amounts to scaling each standard error estimate by about 1.03. This moves the standard error estimates closer to the measured standard deviations, but not meaningfully so. For example, the standard deviation of the estimates of γ in

Panel A is 2.38 times greater than the mean estimated standard error. For the other instruments, the Ferson and Foerster (1994) adjustment fares worse. The adjustment of 1.08 for the largest instrument set (Series 3) is dwarfed in magnitude by the standard deviation of the estimates.

4.2 Goodness-of-Fit Tests

In Table 4, I report the proportion of times the goodness-of-fit test $J_{T,EZ}$ rejects the EZ model at the one, five and ten percent critical values from a $\chi^2(dr\text{-}k)$ distribution. Similar to evidence documented by Kocherlakota (1990b) and Ferson and Foerster (1994), I find that the goodness-of-fit test overrejects the true model. The results, however, are not uniform across the three instrument sets: the Series 1 rejection rates are typically lower and close to their nominal size, while Series 2 and 3 always overreject. Interestingly, this pattern is consistent with the test statistic values observed in Epstein and Zin (1991). In their paper, when either of their first two instruments are used, the test statistic always reject the EZ model with high values of the test statistic, while the third instrument yields low-valued test statistics that do not reject (see Tables 2-4 in Epstein and Zin, 1991).

4.3 Power of the Hypothesis Tests

Table 5 reports the size-adjusted power of the D and Wald hypothesis tests and the goodness-of-fit test $J_{T,TSEU}$ to reject the TSEU null when it is false. By "size-adjusted", I mean the power of the tests is judged relative to empirically determined critical values. This critical value is calculated to be the value out of the 500 simulations that rejects the null x percent of the time (where x is the desired size of the test), when γ is restricted to equal one. To determine the power of the test for a given (α, ρ) pair, I set the TSEU curvature parameter ψ equal to α and vary the value of ρ. I choose to fix ψ equal to α, since this would be the value of the curvature parameter in the i.i.d. case. Fixing ρ and varying α does not meaningfully change the power results.

The large standard deviation in the estimates reported in Table 3 hint that rejection of the $\gamma = 1$ hypothesis will be difficult.   This suspicion is confirmed in Table 5. The power across all tests to reject the TSEU null is very low. In most cases, the power is not much different from the empirical size of the test.

4.4 Results Using Extreme Parameters

In Table 6, I summarize small sample results from simulations using the extreme EZ parameters. Because it produces the best small sample results, I report most of the remaining results using only Instrument Series 1. The biases in the mean and median parameter estimates of γ in Panel A of Table 6 appear to be much lower than their counterparts in Panel A of Table 3. Moreover, the mean standard error is now often much *larger* than the standard deviation of the estimates, while the median standard error remains biased downward. Panel B suggests that the large mean standard errors do not translate to poor size properties for the EZ goodness-of-fit tests.

Most notable is the lack of improvement in the power of the D and $J_{T,TSEU}$ tests. Even when the median estimate of γ is 35.0 ($\alpha = 29.0$, $\rho = 2.0$), the power is only marginally larger than size of the test. The Wald test, on the other hand, demonstrates some power to reject the TSEU null when the difference between α and ρ is large.   For the case when $\alpha = 29.0$ and $\rho = 2.0$, the power of the Wald test at a 5 percent size is 43 percent. Although the Wald test still does not correctly reject in over 50 percent of the cases, the power is substantially better than previous cases.

4.5 Robustness of Results

Table 7 examines the robustness of the above results to variations in sample size, specification of the DGP, choice of instruments and calculation of the weighting matrix. In particular, the following additions are reported in various combinations, along with settings used in previous tables:

- *Variations in sample size*. A sample size of 350 observations roughly corresponds to the number of postwar monthly observations on aggregate consumption available in the U.S. A sample of size of 5000 observations is used to study the convergence properties of the estimators. The sample sizes are listed in the first column of Table 7.

- *Additional instrument sets.* I check the properties of two other instrument sets. The first instrument set contains only a constant (labeled with a 1 in the "Instrument Set" column) and therefore exactly identifies the EZ model. This instrument set also produces unconditional estimates of the EZ model, since no lagged information is used in forming the estimator. The second instrument set is the Hansen (1985) asymptotically optimal estimator described in Section 3.5 and the appendix. Since the optimal instrument is constructed as k linear combinations of the d assets, the optimal estimator also exactly identifies the EZ model.

- *Different techniques for estimating the weighting matrix.* I explore the properties of the iteratively updated (Ferson and Foerster, 1994) and continuously updated (Hansen, Heaton and Yaron, 1996) weighting matrices, described in section 3.6. These variations are identified under the "Weighting Matrix" column.

- *Alternative specifications of the endowment process.* I examine several variations on the original, annual endowment process. First, I generate estimates using the monthly model in equation (8), instead of the annual model in (7). The results using the model in (8) are marked "monthly" in the "Endowment Process" column of Table 7. Second, holding all other parameters in the annual endowment process constant, I increase the magnitude of negative serial dependence in the consumption growth series. I consider two specifications: $Corr(C_t,C_{t-1}) = -0.30$ and $Corr(C_t,C_{t-1}) = -0.50$. These are marked accordingly under the "Endowment Process" column of Table 7. Third, within the framework of the annual DGP, I reduce the contemporaneous correlation between consumption growth and the return on aggregate wealth by increasing the value of the inverse of the EIS parameter ρ to 40.

Except for when noted, all results in Table 7 are derived using the EZ parameter values $\beta = 0.98$, $\alpha = 29.0$ and $\rho = 2.0$. The instrument set identified as "1, $R_{sp,t-1}$" in the "Instrument Set**"** column is the Instrument Series 1 from previous tables. Panel A contains the statistics related to the model parameter estimates, while Panel B reports the size and power of the tests. Note that for comparison purposes, the first row in Panels A and B is taken from Table 6. It is the case that uses 90 observations, Instrument Series 1, 2-Step estimation, annual DGP - taken from Table 6.

Before summarizing the results in Table 7, I should highlight several problems that arose during estimation. First, the numerical optimization routine crashed in roughly 20% of the runs using the optimal instrument set.[13] The crashes apparently resulted from unstable estimates of the Hessian matrix used in the Gauss-Newton search; inversion of the estimated Hessian lead to sudden jumps in estimates of γ and ρ to "illegal" regions. This, in turn, resulted in divide-by-zero or imaginary-number errors. Similar problems arose when the "1" (that is, a constant only) was used. This suggests the problem may have to do with identification of the system rather than use of the optimal instruments. I return to problems of identification in Section 4.6. Second, consistent with the problems reported by Hansen, Heaton and Yaron (1996), I found the continuous-updating estimator often had convergence problems. Although the optimization program did not crash, the iterations would either lead to unreasonably large parameter estimates, or would become "stuck" without converging, at parameter values near the true values. The continuous-updating method also appeared to be more sensitive to starting values of the optimization routine.

Use of the optimal instrument estimator does not greatly improve any of the small sample properties of the estimates or tests; nor does simply increasing the sample size to 350 observations. When I increase the sample size to 350 and draw data from the monthly process (8), the mean estimate of the standard error correctly reflects the dispersion in the parameter estimates. However, the power of three tests does not increase. I introduce the iterative estimator, described by Ferson and Foerster (1994), using 350 observations and the monthly endowment process, since it is under this scenario that Hansen, Heaton and Yaron (1996) find the estimator yields tests with the best size properties. Interestingly, the point and standard error estimates using the iterative estimator are much poorer than the 2-step case, while the $J_{T,EZ}$ statistic rejects too often. The power of the Wald and $J_{T,TSEU}$ tests are slightly higher, though overall, the iterative technique does not appear to improve much on the 2-step estimator.

The continuous-updating estimator produces a mean estimate ρ 14 times its true value and the standard deviation of the estimates of ρ is much higher than the mean value of its standard error. Other than the problem with the estimates of ρ, the estimator performs very well. The two other parameters of the EZ model, and their standard errors, are well estimated. Although the goodness-of-fit $J_{T,EZ}$ test tends to over-reject, the power of tests based on the continuous-updating estimator is high. The power, at a 5 percent size, of the Wald test is 78.5 percent, while the power of the $J_{T,TSEU}$ test is 62 percent.[14]

Increasing the level of first-order negative serial correlation in consumption growth from –0.15 under the original specification to –0.30 does little to change the power of the three test statistics. However, when the level of serial dependence is increased to –0.50, the power of the Wald and $J_{T,TSEU}$ tests increase to 36 and 63 percent, respectively. This result is consistent

Kocherlakota's (1990a) conjecture that moving away from i.i.d. consumption growth aids in separating asset pricing restrictions implied by the EZ and TSEU asset pricing models. However, the magnitude of serial correlation required to obtain the higher power is unrealistic when compared to real-world data.

The EZ marginal rate of substitution depends both on aggregate consumption growth and the return on aggregate wealth, while the TSEU marginal rate of substitution depends only on consumption growth. Therefore, the degree to which a return series generated by the EZ model differs from that from a TSEU model will depend on the variation in aggregate wealth unrelated to the variation in consumption growth. By assuming that the aggregate wealth portfolio is a security whose dividend series is the aggregate consumption process, I may be handicapping the power of the tests.

Indeed, the contemporaneous correlation between the artificial consumption growth and aggregate wealth return series is high, apparently leaving little independent explanatory power for the aggregate wealth variable. The annual endowment process, coupled with the EZ parameters $\alpha = 29.0$ and $\rho = 2.0$, produces a contemporaneous correlation between consumption growth and the return on aggregate wealth of 0.988, while the combination of $\alpha = 29.0$ and $\rho = 29.0$ generates a correlation of 0.736. This compares to a measured correlation of 0.372 between the U.S. aggregate consumption growth and S&P 500 stock market data used in Mehra and Prescott (1985) (see Kocherlakota, 1990b, Table 2). Hall (1988) argues that the low correlation between consumption growth and security returns requires a low intertemporal rate of substitution (high ρ). When ρ is set to 40.0 in the context of the annual DGP, the contemporaneous correlation between consumption growth and the aggregate wealth return falls to 0.695.[15] As is evident in the row marked "annual ($\rho = 40.0$)" of Panel B in Table 7, allowing for the return on aggregate wealth vary more independently of consumption growth does not improve the power of the tests.

I attempted to reduce the correlation between consumption growth and the return on aggregate wealth and ran simulations using two other approaches. First, I redefined the aggregate wealth security to be a claim on the S&P 500 dividend, instead of consumption. Such a change breaks the most obvious association between consumption growth and aggregate wealth, although equilibrium returns will still be related to consumption growth through its influence on the marginal rate of substitution. Second, I added noise to the original consumption growth series to represent an additional random endowment to agent wealth, such as labor income, not reflected in consumption. The noise was added as a mean-preserving spread to the state values of consumption growth. Neither of the two corrections greatly reduced the contemporaneous correlation between consumption growth and the aggregate wealth portfolio, nor improved the power of the tests. Therefore, the results are not reported in Table 7.

At 5000 observations, the two-step and iterative procedures produce similar estimates and have high power to reject the TSEU null. However, the iterative procedure continues to reject the EZ model using $J_{T,EZ}$, while the two-step estimator tends to underreject.

4.6 EZ Parameters are Poorly Identified

Why do most of the tests studied here have such low power? Analysis of the estimated variance-covariance matrix $(G_T{}' W_T G_T)^{-1}$reveals the parameters of the EZ model to be poorly identified. Table 8 reports average values of what Belsley (1991) terms *scaled condition indexes* and *variance decomposition proportions* of $(G_T{}' W_T G_T)^{-1}$. The *j*th scaled condition index is defined to be the square root of the ratio of the maximum eigenvalue of $(G_T{}' W_T G_T)^{-1}$ to its *j*th eigenvalue, taken after the columns of the matrix have been scaled to have unit length. The indexes gauge the number of near linear dependencies existing among the columns of the

Jacobian matrix G_T (weighted by the elements of W_T). Collinearity among two or more columns indicates that common variation exist amongst those columns. The common variation leads to imprecise parameter estimates. The (*k*, *j*)th variance decomposition proportion is the proportion of the *k*th parameter variance estimate explained by the *j*th condition index. According to Belsley (1991), two or more high decomposition proportions associated with a high-valued condition index indicates a poorly identified model. Belsley (1991) argues from experimental evidence that a scaled condition index greater than 10 indicates the presence of weak collinearity, while a scaled index greater than 30 indicates collinearity strong enough to adversely affect the preciseness of parameter estimates.

The first panel of Table 8 reports the average value of the condition indexes and variance decomposition proportions from 500 simulations using Instrument Series 1, the annual endowment process specified in (7) and the parameters $\alpha = 0.80$ and $\rho = 0.80$. The highest condition index is above the cutoff value of 30 (79.612), suggesting that presence of one strong linear dependency. This one dependency explains much of the estimated variance of *all* three parameters: 93 percent of the estimated variance of β_T, 98 percent of the variance in ρ_T and 55 percent of the variance in γ_T. For the case $\alpha =29.0$, $\rho = 29.0$ in Panel B, the highest condition index is below 10 (7.685), suggesting that collinearity should not be a problem for this parameterization. However, when ρ is set to 2.0 in Panel C, one strong linear dependency (condition index = 238.687) again explains a large proportion of the variance of all three parameter estimates. Thus, even with large differences between the RRA parameter α and the inverse EIS parameter ρ , the model may be poorly identified. The last two panels of Table 8 employ the extreme parameterizations ($\alpha = 29.0$; $\rho = 29.0, 2.0$), but increase the serial dependence in consumption growth to –0.50. Consistent with the higher power documented in

Table 7 for the Corr(C_t,,C_{t-1}) = -0.50 case, the highest condition number and variance decomposition proportions drop in both panels.

The other results in Table 8 also accord well with the results from earlier tables. For instance, the standard deviation of the estimates of γ across 500 simulations in the $\alpha = \rho = 0.80$ case is 2.5 times greater than the mean estimate of γ (Table 3, Panel A), while for the $\alpha = \rho = 29.0$ case, the standard deviation is only 0.75 of the mean estimate (Table 6, Panel A). Data generated for high α and ρ produce more precise estimates of γ.

Collinearity problems arise in linear models solely as a result of correlation among the columns of the data matrix of explanatory variables. By contrast, collinearity problems can occur in the models studied here even when the data are well-conditioned. To see this, consider the case where the instrument set z_t is set to a column vector of ones, and $R_{j,t+1} = R_{w,\, t+1}$. The matrix $G_T(b_T)$ can be represented as

$$\left[\frac{\gamma_T}{\beta_T}\frac{1}{T}\sum_{t=1}^{T}X_{j,t+1} \quad \frac{\gamma_T}{\rho_T}\frac{1}{T}\sum_{t=1}^{T}X_{j,t+1}\bullet\left[\ln(X_{j,t+1})-\ln(\beta_T)-\ln(R_{w,t+1})\right] \quad \frac{\gamma_T}{\rho_T}\frac{1}{T}\sum_{t=1}^{T}X_{j,t+1}\bullet\ln(X_{j,t+1})\right],$$

$$\text{where } X_{j,t+1} \equiv \left(\beta_T\left(\frac{C_{t+1}}{C_t}\right)^{-\rho_T}R_{w,t+1}\right)^{\gamma_T}\frac{R_{j,t+1}}{R_{w,t+1}}. \qquad (10)$$

The columns of this matrix will be correlated. The value $X_{j,t+1}$ is defined to be the product of the *(t+1)*th observation on the marginal rate of substitution times the *(t+1)*th return on asset *j*. Each of the three columns in (10) is a weighted average of the $X_{j,t+1}$ (t = 1,2, . . ., T) and differ only by a constant scale factor and by how $X_{j,t+1}$ is weighted in the average. The first column is scaled by (γ_T/β_T) and weighted by 1/T. The second column is scaled by (γ_T/ρ_T) and weighted by (1/T)[ln(X_t) – ln(β_T) – ln($R_{w,t+1}$)] and the third column is scaled by ($1/\gamma_T$) and weighted by (1/T)ln($X_{j,t+1}$). The degree of correlation between these columns depends entirely on the correlation between these weights. As noted in Section 3.3, when either γ_T or ρ_T go to zero, the identification problem increases.

Adding additional instruments to z_t can either improve or further degrade the Jacobian, depending on the correlation between the added instrument and the columns in (10). Each additional instrument simply adds 3 rows (j = 1, 2, 3) to (10). The three new rows will be the average of the cross-product between the elements of the first three rows and the new element in z_t.

5. Summary and Conclusion

This paper investigates the small sample properties of GMM estimators and tests of the Epstein and Zin (1989, 1991) asset pricing model. The estimated asymptotic standard errors associated with the parameter estimates are biased downward and tests of each model against an unspecified alternative that use asymptotic chi-squared critical values can over-reject. Therefore, without proper adjustment to the standard errors or test statistics, a researcher may incorrectly reject the TSEU model in favor of the EZ model. Adjusting for these biases, hypothesis tests based on the first order conditions from an EZ agent's maximization problem have little power over the null model that constrains the RRA and the reciprocal EIS parameters to be equal. Expressed in relative terms, the Wald test appears to have greater power than the Newey and West (1987) D-test or Hansen's (1982) goodness-of-fit test of the TSEU model. Still, at its best the Wald test rejects less than 50 percent of the time.

These results are robust to increasing the number of observations to 350, to use of an iterative estimator in place of the two-step estimator and to changes in the endowment process. Moreover, use of an instrument set that asymptotically minimizes the covariance matrix of the estimators does not greatly increase the power of the tests, suggesting that the use of "poor" instruments is not the cause of low power. Employing the continuous-updating estimator proposed by Hansen, Heaton and Yaron (1996) strengthens the power of the Wald test further and increases the power of the $J_{T,TSEU}$ test (the D-test is not examined for this case), at least for the case where the differences between the EZ model parameters are chosen to be large. Using only 90 observations, the power of the Wald test is nearly 80 percent and the power of the $J_{T,TSEU}$ is over 60 percent.

I conjecture that the low power of the tests stems from the nature of the nonlinear Euler restrictions. The functional form of the restrictions induces correlation among the columns of the Jacobian matrix. These dependencies degrade the preciseness of the parameter estimates much in the way as collinear data in a linear model.

Appendix

Generating Artificial State Returns

Let s_t be the t-th state drawn from the set S of states. The Tauchen-Hussey approximation yields a vector of state values λ_j and ζ_j, j = 1, 2, …, S, a vector of stationary probabilities Π_i, i = 1, 2, .. S and a set of probability weights $\pi_{ij} = \Pr\{s_{t+1} = \lambda_j, \zeta_j \vdash s_t = \lambda_i, \zeta_i\}$, i,j = 1, 2, .. , S. To obtain state values of asset returns, I define $x_{w,t} = d_{w,t}/d_{w,t-1} = \lambda_t$ to be the growth rate in a dividend paid on the aggregate wealth portfolio, which is assumed equal to growth rate in aggregate consumption and $v_{,w,t} = p_{w,t}/d_{w,t}$ to be the price-dividend ratio from the claim on aggregate wealth. Then, the finite-Markov representation of the Euler equation (2), implies that S equations can be set up of the form

$$v_{w,i}^{\frac{1-\alpha}{1-\rho}} = \sum_{j=1}^{S} \pi_{ij} \left[\beta^{\frac{1-\alpha}{1-\rho}} \lambda_j^{1-\alpha} (1 + v_{w,j})^{\frac{1-\alpha}{1-\rho}} \right], i = 1,2,\ldots,S.. \qquad \text{(A.1)}$$

Let v_w^* represent the S-vector of solutions to (A.1). For a fixed endowment process, v_w^* will differ as the subjective discount factor β, RRA parameter α and reciprocal EIS parameter ρ are varied. For fixed values of β, α and ρ, I obtain three sets of state-returns from the solutions v^*_w. The return on aggregate wealth over states i and j is given by,

$$R_{w,ij} = \frac{1 + v^*_{w,j}}{v^*_{w,i}} \lambda_j.$$

The price in state i of a bond paying unity for sure next period is

$$\frac{1}{R_{fi}} = \beta^{\frac{1-\alpha}{1-\rho}} \sum_{j=1}^{S} \pi_{ij} \lambda_j^{(-\alpha)} \left(\frac{1 + v^*_{w\,j}}{v^*_{w,i}} \right)^{\frac{(\rho-\alpha)}{(1-\rho)}} .$$

A analogous solution to (A.1) is obtained when the return on the simulated S & P 500 dividend stream is substituted for the claim on aggregate wealth. The state returns are given by

$$R_{sp,ij} = \frac{1 + v^*_{sp,j}}{v^*_{sp,i}} - \zeta_j \,,$$

where $v^*_{sp,i}$ represents the price-dividend ratio of the S & P 500 index in state i.

Drawing a Random Sample of Observations

Let R^θ be the number of Monte Carlo experiments conducted for a fixed set of parameters, $\theta = \{\beta, \alpha, \rho)$ and let $y^\theta = (y^\theta(1), y^\theta(2), \ldots, y^\theta(N))'$ be an $S \times (S+2)$ realization matrix, $y^\theta(n)$ is a $1 \times (S+2)$ vector of the market portfolio returns, the risk-free return and consumption growth realization in state n. For each experiment r^θ, t = 1, 2, . . ., T realizations are drawn using the following algorithm:

(1) Draw u_0 from a U[0,1] distribution. Let the initial state, n0 be the smallest number such that

$\Pi(1) + \Pi(2) + \ldots + \Pi(n0) \geq u_0$

where $\Pi(i)$ is the stationary probability of being in state i.

(2) Let n' (= n_0 for initial draw) be the previous state and n'' be the next state to be drawn. Draw u'' from U[0,1] and let n'' be the smallest number such that

$\pi(n',1) + \pi(n',2) + \ldots + \pi(n',n'') \geq u''$

where $\pi(i,j)$ is the probability of moving to state j, given you are in state i.

(3) From y^θ, choose $R_{sp}(n',n'')$, $R_e(n',n'')$, $R_f(n')$ and $\lambda(n'')$.

(4) Set n' = n''. Return to step 2 until t = T.

Calculating the Optimal Instrument Set

Tauchen (1986) shows that the GMM estimator b_T is asymptotically equivalent to an estimator $\bar{b}_T$ which minimizes the quadratic $\bar{g}_{HT}(x_{t+1}, b)'\bar{g}_{HT}(x_{t+1}, b)$, where

$$\bar{g}_{HT}(x_{t+1}, b) = \frac{1}{T}\sum_{t=1}^{T} H_t u(x_{t+1}, b), \qquad \text{(A.2)}$$

and H_t is a $k \times d$ "instrument" matrix formed based on information available at time t. This alternative interpretation of the GMM estimator is analogous to interpreting the generalized least squares (GLS) estimator as a simple instrumental variables estimator with instrument matrix equal to the product of the inverse variance-covariance and X matrices (see Davidson and MacKinnon, 1993, p. 295).

With GMM estimation, H_t is a linear combination of the product of the estimated gradient and weighting matrices, $G_T = \partial g_T(b_T)/\partial b_T$, W_T and the instruments in z_t. Hansen (1985) demonstrates that the choice of the instrument matrix that minimizes, over all feasible choices of H_t, the asymptotic variance-covariance matrix of the GMM estimator is given by

$$\begin{aligned} H_t^* &= G^{*}{}'V^{*-1}, \text{where} \\ G^* &= E_t\left[\frac{\partial u(x_{t+1}, b_0)}{\partial b_0}\right] \\ V^* &= E_t\left[u(x_{t+1}, b_0)u(x_{t+1}, b_0)'\right]. \end{aligned}$$

For each draw of data, I implement the optimal instruments by first calculating H_t^*, t = 1, . . . T using the parameters of the DGP. I then estimate (A.2) setting $H_t = H_t^*$.

Footnotes

[1] The Epstein and Zin (1989) model relaxes the assumption of state separability. Other generalizations of the TSEU model allow for time non-separabilities. See Dunn and Singleton (1986), Constantinides (1990), Ferson and Constantinides (1991) and Heaton (1995).

[2] Epstein and Zin (1989) extend the work of Kreps and Porteus (1978), who generalize von Nueman-Morgenstern utility theory to allow agents to have preferences over the timing of the resolution of uncertainty. For instance, an agent who can enter a coin toss to determine a next-period payoff may prefer for the coin to be flipped today, rather than next period. Esptein and Zin (1989) extend the Kreps-Porteus preferences to an infinite horizon framework and show that preferences so constructed maintain stationarity of preferences and time consistency of consumption plans.

[3] When $\alpha = \rho$, m_{t+1} reduces to the marginal rate of substitution for an agent endowed with TSEU preferences,

$$E_t\left[\beta \frac{\tilde{c}_{t+1}}{c_t} \tilde{R}_{j,t+1}\right] = 1, \; j = 1, 2, \ldots, N.$$

Under the TSEU model, the curvature parameter, ψ, can be interpreted as *either* the relative risk aversion parameter *or* the reciprocal of the elasticity of intertemporal substitution parameter. As noted by Hall (1988) and Lucas (1978), there is no economic reason for believing that an agent's attitude toward risk be related to the agent's preference for certain consumption now versus certain consumption streams in the future.

[4] Hansen (1982) assumes that the process $\{x_t\}$ is stationary and ergodic, that the second moments of $u(x_{t+1}, b_0)$ and z_t exist and are finite, that $g(b)$ is continuous in b and that W_T approaches a constant, symmetric, positive-definite weighting matrix, as T grows large.

[5] Tauchen (1986) and Tauchen and Hussey (1991) explore the properties of Monte Carlo simulations based on finite-state approximations to endowment processes such as (7) and (8). They demonstrate that the distribution of the draws from an approximating Markov process converges to its underlying continuous autoregression as the number of states becomes large. Tauchen (1986) finds that allowing each endowment variable to take on one of eight possible

values (or with two endowment variables, 64 states of nature) satisfactorily approximates the continuous VAR. I follow Kocherlakota (1990b) and report results using four possible realizations of each endowment variable, implying 16 possible states. The appendix also describes the procedure for obtaining each realization of data.

[6] See Epstein and Zin (1991), Table 3.

[7] Similar results are documented by Kandel and Stambaugh (1991).

[8] See, for example, Shanken (1990), Ferson and Harvey (1991) and Harvey (1990).

[9] Starting the search algorithm at the true population values could bias the small sample properties of the estimators. However, for the bulk of the tests, I find that choice of the starting values does not alter the estimates. I did find evidence that the continuous-updating estimator is sensitive to starting values.

[10] For the iterative simulations reported here, convergence of the estimated weighting matrix is achieved when the largest absolute change in an element of the weighting matrix is less than 0.01.

[11] To allow for the relative risk aversion parameter α to approach one (which yields logarithmic risk preferences), I follow Epstein and Zin and divide the error equation corresponding the return on aggregate wealth, $u_e(x_{t+1}, b_0) = \beta\left(\left(\frac{c_{t+1}}{c_t}\right)^{-\rho}\right)^{\gamma} R_{we,t+1}{}^{\gamma} - 1$, by γ. As γ approaches one, $u_e(x_{t+1}, b_0)/\gamma$ approaches an estimable expression that is linear in the logarithm of the variables. See Epstein and Zin (1991), equation (19).

[12] For each draw of data, I calculate the standard error of α using the "delta method". Let $f(\theta) \equiv \alpha = 1 - \gamma(1-\rho)$ and $V(\theta)$ be the variance covariance matrix with $\theta = (\rho \;\; \gamma)'$. Then the standard error of α is the square root of $\frac{\partial f}{\partial \theta'} V(\theta) \frac{\partial f}{\partial \theta}$.

[13] A similar problem arising from using estimates of the optimal instrument set is reported in Tauchen (1986).

[14] The D-test is not calculated since it requires that the parameters under the TSEU be estimated using the weighting matrix from the EZ estimates.

[15] The algorithm used to solve the nonlinear state-price equations did not converge for values of ρ greater than 40.

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Table 1
Summary Information on Endowment Variables and Security Returns from Monte Carlo Simulations

Using 500 Draws of 90 Observations from Annual DGP

Panel A: Mean slope and standard error estimates and standard deviation of estimates from annual VAR(2) regressions of consumption and dividend growth.

		Estimates				
Variable		Intercept	ζ_{t-1}	λ_{t-1}	Var(ε)	Cov(ε)
ζ_t	Mean point estimate	0.004	0.104	0.421	0.013	0.002
	Mean std error estimate	0.013	0.107	0.367		
	Std dev of estimates	0.013	0.110	0.390		
λ_t	Mean point estimate	0.021	0.017	-0.162	0.001	0.002
	Mean std error estimate	0.004	0.032	0.108		
	Std dev of estimates	0.004	0.032	0.111		

Table 1 (continued)
Summary Information on Endowment Variables and Security Returns from Monte Carlo Simulations
Using 500 Draws of 90 Observations from Annual DGP

Panel B: Average sample mean, standard deviation and equity premium of simulated securities returns using parameter values from Epstein and Zin (1991).

α. ρ		R_e	R_{sp}	R_f	Equity Premium
0.80, 0.80	Mean of estimates	0.036	0.036	0.035	0.001
	Std dev of estimates	0.036	0.147	0.050	
0.80, 5.20	Mean of estimates	0.124	0.125	0.122	0.003
	Std dev of estimates	0.064	0.163	0.030	
1.35, 1.35	Mean of estimates	0.047	0.050	0.045	0.005
	Std dev of estimates	0.387	0.149	0.010	
1.35, 5.20	Mean of estimates	0.123	0.124	0.120	0.004
	Std dev of estimates	0.064	0.164	0.030	

Table 1 (continued)
Summary Information on Endowment Variables and Security Returns from Monte Carlo Simulations Using 500 Draws of 90 Observations from Annual DGP

Panel C: Average sample mean, standard deviation and equity premium of simulated securities returns using extreme parameter values.

α. ρ		R_e	R_{sp}	R_f	Equity Premium
29.0, 29.0	Mean of estimates	0.204	0.246	0.059	0.176
	Std dev of estimates	0.247	0.297	0.159	
29.0, 13.7	Mean of estimates	0.109	0.153	0.030	0.120
	Std dev of estimates	0.120	0.194	0.072	
29.0, 2.0	Mean of estimates	0.045	0.089	0.010	0.079
	Std dev of estimates	0.041	0.154	0.010	
2.0, 29.0	Mean of estimates	0.738	0.741	0.713	0.016
	Std dev of estimates	0.324	0.385	0.257	
Actual	Mean of estimates	0.000	0.070	0.010	0.059
	Std dev of estimates	0.000	0.165	0.055	

Table 2
Summary of Estimates of Instrument Autoregressions and Regressions of Returns and Consumption Growth Using 500 Draws of 90 Observations from Annual DGP

Panel A: Mean point and t-ratio estimates from instrument autoregressions using various values of the RRA parameter α and EIS parameter ρ.

Consumption Growth

Variable		Intercept	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	Adj-R^2
λ	Mean point estimate	1.220	-0.145	-0.012	-0.012	-0.020	-0.008	0.062
	Mean t-ratio	4.800	-1.474	-0.118	-0.117	-0.207	-0.087	

$\beta = 0.98$, $\alpha = 0.80$, $\rho = 0.80$

Variable		Intercept	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	Adj-R^2
R_{sp}	Mean point estimate	1.125	-0.022	-0.021	-0.011	-0.015	-0.014	0.049
	Mean t-ratio	4.733	-0.229	-0.225	-0.118	-0.160	-0.162	
R_f	Mean point estimate	1.330	-0.214	-0.029	-0.014	-0.023	-0.005	0.091
	Mean t-ratio	4.852	-2.164	-0.284	-0.124	-0.223	-0.051	

$\beta = 0.98$, $\alpha = 29.0$, $\rho = 29.0$

Variable		Intercept	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	Adj-R^2
R_{sp}	Mean point estimate	1.986	-0.443	-0.096	-0.023	-0.022	-0.008	0.216
	Mean t-ratio	4.755	-4.312	-0.840	-0.198	-0.207	-0.097	
R_f	Mean point estimate	1.393	-0.220	-0.034	-0.020	-0.022	-0.019	0.099

Mean t-ratio	4.667	-2.132	-0.310	-0.192	-0.214	-0.195

Table 2 (continued)
Summary of Estimates of Instrument Autoregressions and Regressions of Returns and Consumption Growth Using 500 Draws of 90 Observations from Annual DGP

$\beta = 0.98$, $\alpha = 29.0$, $\rho = 2.0$

Variable		Intercept	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	Adj-R^2
R_{sp}	Mean point estimate	1.177	-0.029	-0.014	-0.007	-0.020	-0.011	0.053
	Mean t-ratio	4.688	-0.300	-0.147	-0.078	-0.201	-0.118	
R_f	Mean point estimate	1.318	-0.215	-0.037	-0.016	-0.024	-0.014	0.092
	Mean t-ratio	4.889	-2.184	-0.365	-0.147	-0.241	-0.136	

Panel B: Mean slope and t-ratio estimates from regressions of returns and consumption growth on instruments using various values of the RRA parameter α and EIS parameter ρ.

$\beta = 0.98$, $\alpha = 0.80$, $\rho = 0.80$

Variable		Constant	$R_{sp,t-1}$	λ_{t-1}	$R_{f,t}$	Adj-R^2
R_{sp}	Mean point estimate	1.076	0.000	-0.150	0.110	0.031
	Mean t-ratio	0.298	0.001	-0.303	0.034	
R_e	Mean point estimate	1.084	0.011	-0.141	0.067	0.035
	Mean t-ratio	1.329	0.399	-1.248	0.100	
λ	Mean point estimate	1.107	0.014	-0.172	0.071	0.051
	Mean t-ratio	1.351	0.498	-1.519	0.086	

Table 2 (continued)
Summary of Estimates of Instrument Autoregressions and Regressions of Returns and Consumption Growth Using 500 Draws of 90 Observations from Annual DGP

$\beta = 0.98$, $\alpha = 29.0$, $\rho = 29.0$

Variable		Constant	$R_{sp,t-1}$	λ_{t-1}	$R_{f,t}$	Adj-R^2
R_{sp}	Mean point estimate	9.764	0.609	-8.411	-0.667	0.412
	Mean t-ratio	6.378	2.547	-5.442	-2.328	
R_e	Mean point estimate	9.491	0.605	-8.185	-0.661	0.545
	Mean t-ratio	3.609	0.937	-2.715	0.025	
λ	Mean point estimate	1.262	0.016	-0.243	-0.015	0.056
	Mean t-ratio	3.446	0.765	-1.521	0.102	

$\beta = 0.98$, $\alpha = 29.0$, $\rho = 2.0$

Variable		Constant	$R_{sp,t-1}$	λ_{t-1}	$R_{f,t}$	Adj-R^2
R_{sp}	Mean point estimate	1.526	0.020	-0.346	-0.105	0.036
	Mean t-ratio	0.992	0.160	-0.654	-0.070	
R_e	Mean point estimate	1.368	0.028	-0.355	0.008	0.095
	Mean t-ratio	3.609	0.937	-2.715	0.025	
λ	Mean point estimate	1.150	0.013	-0.175	0.033	0.051
	Mean t-ratio	3.446	0.765	-1.521	0.102	

Table 3
Summaries of Estimates of EZ Parameters from Monte Carlo Simulations Using Parameter Values from Epstein and Zin (1991) and 500 Draws of 90 Observations and Annual DGP

Panel A: $\beta = 0.98$, $\alpha = 0.80$, $\rho = 0.80$, ($\gamma = 1.000$)

Instrument Series		β	α	ρ	γ
Series 1	Mean point	0.980	-0.687	0.476	53.600
	Median point	0.983	0.798	0.963	21.411
	Mean std error	0.032	5.420	4.102	55.521
	Median std error	0.001	3.807	0.063	20.030
	Std dev of estimates	0.050	6.324	8.771	132.856
Series 2	Mean point	0.984	-0.792	1.009	29.806
	Median point	0.982	0.011	0.920	15.082
	Mean std error	0.002	1.223	0.073	3.029
	Median std error	0.001	1.015	0.036	1.802
	Std dev of estimates	0.006	5.041	0.282	49.246
Series 3	Mean point	0.984	-0.483	1.041	26.063
	Median point	0.982	0.225	0.931	15.546
	Mean std error	0.002	0.986	0.071	1.901
	Median std error	0.001	0.820	0.032	1.201
	Std dev of estimates	0.005	4.986	0.284	34.086

Table 3 (continued)

Summaries of Estimates of EZ Parameters from Monte Carlo Simulations Using Parameter Values from Epstein and Zin (1991) and 500 Draws of 90 Observations and Annual DGP

Panel B: $\beta = 0.98$, $\alpha = 0.80$, $\rho = 5.20$, ($\gamma = -0.0476$)

Instrument Series		β	α	ρ	γ
Series 1	Mean point	0.917	0.558	0.984	-1.201
	Median point	0.927	1.614	2.157	-1.248
	Mean std error	0.032	3.532	4.226	1.643
	Median std error	0.007	2.892	0.469	1.060
	Std dev of estimates	0.071	5.489	12.167	6.087
Series 2	Mean point	0.934	-0.069	2.279	-0.877
	Median point	0.937	-0.347	2.809	-1.524
	Mean std error	0.008	1.512	1.103	0.300
	Median std error	0.006	1.386	0.336	0.221
	Std dev of estimates	0.048	6.548	9.929	3.664
Series 3	Mean point	0.930	-0.160	2.332	-1.050
	Median point	0.934	-0.336	2.605	-1.790
	Mean std error	0.007	1.224	0.361	0.239
	Median std error	0.005	1.172	0.283	0.206
	Std dev of estimates	0.026	6.898	1.619	4.016

Table 3 (continued)
Summaries of Estimates of EZ Parameters from Monte Carlo Simulations Using Parameter Values from Epstein and Zin (1991) and 500 Draws of 90 Observations and Annual DGP

Panel C: $\beta = 0.98$, $\alpha = 1.35$, $\rho = 1.35$, ($\gamma = 1.000$)

Instrument Series		β	α	ρ	γ
Series 1	Mean point	0.974	2.239	0.705	33.011
	Median point	0.975	2.022	1.058	13.681
	Mean std error	0.006	3.917	0.554	20.063
	Median std error	0.002	2.419	0.102	6.506
	Std dev of estimates	0.060	5.320	5.831	64.134
Series 2	Mean point	0.973	3.202	0.863	18.514
	Median point	0.977	2.740	1.156	11.483
	Mean std error	0.004	1.286	0.319	2.041
	Median std error	0.001	0.981	0.051	1.247
	Std dev of estimates	0.038	4.899	3.318	25.205
Series 3	Mean point	0.974	2.830	1.002	18.483
	Median point	0.976	2.218	1.125	11.853
	Mean std error	0.001	0.971	0.086	1.229
	Median std error	0.001	0.787	0.047	0.933
	Std dev of estimates	0.006	5.090	0.356	25.223

Table 3 (continued)
Summaries of Estimates of EZ Parameters from Monte Carlo Simulations Using Parameter Values from Epstein and Zin (1991) and 500 Draws of 90 Observations and Annual DGP

Panel D: $\beta = 0.98$, $\alpha = 1.35$, $\rho = 5.20$, ($\gamma = 0.0833$)

Instrument Series		β	α	ρ	γ
Series 1	Mean point	0.923	1.729	0.977	0.602
	Median point	0.925	2.333	2.107	0.827
	Mean std error	0.026	3.225	4.145	1.622
	Median std error	0.007	2.638	0.458	1.057
	Std dev of estimates	0.090	5.502	14.823	5.001
Series 2	Mean point	0.939	1.929	2.831	0.745
	Median point	0.940	2.189	2.969	1.279
	Mean std error	0.008	1.539	0.435	0.322
	Median std error	0.006	1.371	0.341	0.232
	Std dev of estimates	0.031	6.850	1.843	3.678
Series 3	Mean point	0.935	1.923	2.611	0.688
	Median point	0.936	2.015	2.685	1.594
	Mean std error	0.006	1.143	0.347	0.220
	Median std error	0.005	1.068	0.289	0.188
	Std dev of estimates	0.023	6.874	1.318	3.822

Table 4
Proportion of Rejections of Hansen's Goodness-of-fit Test ($J_{T,EZ}$)
at Nominal Sizes Using Parameter Values from Epsein and Zin (1991),
and 500 Draws of 90 Observations from Annual DGP

Panel A: $\beta = 0.98$, $\alpha = 0.80$, $\rho = 0.80$, ($\gamma = 1.000$)

Instrument Series	Median $J_{T,EZ}$	Nominal Size 0.010	0.050	0.100
Series 1	1.338	0.002	0.020	0.034
Series 2	9.433	0.266	0.398	0.464
Series 3	14.962	0.292	0.430	0.512

Panel B: $\beta = 0.98$, $\alpha = 0.80$, $\rho = 5.20$, ($\gamma = -0.0476$)

Instrument Series	Median $J_{T,EZ}$	Nominal Size 0.010	0.050	0.100
Series 1	2.149	0.016	0.048	0.102
Series 2	16.435	0.492	0.650	0.704
Series 3	22.588	0.532	0.708	0.794

Table 4 (continued)
Proportion of Rejections of Hansen's Goodness-of-fit Test ($J_{T,EZ}$)
at Nominal Sizes Using Parameter Values from Epsein and Zin (1991),
and 500 Draws of 90 Observations from Annual DGP

Panel C: $\beta = 0.98$, $\alpha = 1.35$, $\rho = 1.35$, ($\gamma = 1.000$)

Instrument Series	Median $J_{T,EZ}$	Nominal Size 0.010	0.050	0.100
Series 1	1.940	0.106	0.130	0.146
Series 2	9.912	0.310	0.420	0.480
Series 3	17.778	0.390	0.528	0.608

Panel D: $\beta = 0.98$, $\alpha = 1.35$, $\rho = 5.20$, ($\gamma = 0.0833$)

Instrument Series	Median $J_{T,EZ}$	Nominal Size 0.010	0.050	0.100
Series 1	2.323	0.014	0.061	0.108
Series 2	15.530	0.454	0.626	0.692
Series 3	23.118	0.536	0.684	0.774

Table 5

Size-Adjusted Power of Tests of TSEU Null Against EZ Alternatives Using Parameter Values from Epstein and Zin (1991) and 500 Draws of 90 Observations from Annual DGP

Panel A: $\beta = 0.98$, $\alpha = 0.80$, $\rho = 5.20$, ($\gamma = -0.0476$)

Instrument Series	Test	Median Statistic	Empirical Size 0.010	0.050	0.100
Series 1	D	1.369	0.066	0.156	0.234
	Wald	12.232	0.064	0.260	0.344
	$J_{T,TSEU}$	3.784	0.016	0.056	0.122
Series 2	D	104.913	0.004	0.044	0.122
	Wald	231.226	0.004	0.062	0.128
	$J_{T,TSEU}$	12.680	0.050	0.126	0.186
Series 3	D	230.653	0.000	0.048	0.134
	Wald	366.402	0.000	0.046	0.134
	$J_{T,TSEU}$	20.760	0.016	0.126	0.212

Table 5 (continued)

Size-Adjusted Power of Tests of TSEU Null Against EZ Alternatives Using Parameter Values from Epstein and Zin (1991) and 500 Draws of 90 Observations from Annual DGP

Panel B: $\beta = 0.98$, $\alpha = 1.35$, $\rho = 5.20$, ($\gamma = 0.0833$)

Instrument Series		Median Statistic	Empirical Size 0.010	0.050	0.100
Series 1	D	1.062	0.000	0.019	0.101
	Wald	5.087	0.005	0.007	0.068
	$J_{T,TSEU}$	3.923	0.000	0.000	0.009
Series 2	D	48.972	0.006	0.032	0.110
	Wald	105.287	0.004	0.026	0.076
	$J_{T,TSEU}$	12.832	0.028	0.062	0.122
Series 3	D	144.410	0.000	0.034	0.082
	Wald	187.306	0.000	0.038	0.084
	$J_{T,TSEU}$	20.144	0.026	0.088	0.184

Table 6
Summaries of Parameter Estimates, Nominal Size and Power from Monte Carlo Simulations Using "Extreme" Parameter Values, Instrument Series 1 and 500 draws of 90 observations from Annual DGP

Panel A: Summary of parameter estimates

		Parameters			
α, ρ (γ)		β	α	ρ	γ
29.0, 29.0 (1.0)	Mean point	6.612	32.250	43.362	1.512
	Median point	0.955	30.250	23.711	1.184
	Mean std error	427.997	143.170	225.013	0.207
	Median std error	0.035	6.574	3.324	0.186
	Std dev of estimates	97.093	15.071	154.327	1.127
29.0, 13.7 (2.2)	Mean point	1.015	32.674	16.489	3.579
	Median point	0.973	30.951	10.710	2.903
	Mean std error	0.841	12.545	16.546	0.534
	Median std error	0.020	6.048	1.634	0.393
	Std dev of estimates	0.632	12.489	27.024	3.482
29.0, 2.0 (28.0)	Mean point	0.993	32.721	4.436	71.366
	Median point	0.979	29.658	1.571	42.452
	Mean std error	1.467	6297.152	212.389	10.340
	Median std error	0.002	5.359	0.099	4.743
	Std dev of estimates	0.114	20.169	14.259	97.635

Table 6 (continued)
Summaries of Parameter Estimates, Nominal Size and Power from Monte Carlo Simulations Using "Extreme" Parameter Values, Instrument Series 1 and 500 draws of 90 observations from Annual DGP

Panel B: Proportion of rejections of Hansen's goodness-of-fit $J_{T,EZ}$ test at nominal sizes

α, ρ (γ)	Median TJ_T	Nominal Size 0.010	0.050	0.100
29.0, 29.0 (1.0)	3.710	0.112	0.224	0.312
29.0, 13.7 (2.2)	2.491	0.050	0.128	0.194
29.0, 2.0 (28.0)	1.933	0.040	0.086	0.136

Panel C: Size-adjusted power of tests of TSEU null against EZ alternatives

α, ρ (γ)	Test	Median Statistic	Empirical Size 0.010	0.050	0.100
29.0, 13.7 (2.2)	D	18.174	0.014	0.094	0.160
	Wald	28.187	0.002	0.128	0.208
	$J_{T,TSEU}$	6.178	0.012	0.042	0.100
29.0, 2.0 (28.0)	D	20.782	0.008	0.040	0.082
	Wald	113.506	0.036	0.430	0.554
	$J_{T,TSEU}$	5.721	0.020	0.060	0.124

Table 7
Robustness of Monte Carlo Simulation Results to Changes in Sample Size, DGP, Instrument Set and Weighting Matrix Using "Extreme" Parameter Value α = 29 and ρ = 2 (unless specified otherwise).

Panel A: Summary of parameter estimates

Sample Size	Instrument Set	Weighting Matrix	Endowment Process	Mean Point α	Mean Std Error α	Std Dev of Estimates α	Mean Point ρ	Mean Std Error ρ	Std Dev of Estimates ρ	Mean Point γ	Mean Std Error γ	Std Dev of Estimates γ
90	1, $R_{sp,t-1}$	2-Step	Annual	32.721	6297.150	20.169	4.436	212.389	14.529	71.366	10.340	97.635
90	1	2-Step	Annual	32.699	1701.145	12.882	3.065	52.908	1.103	41.947	1351.513	98.349
90	Optimal	2-Step	Annual	25.957	1135.918	7.734	1.941	8.546	0.228	27.587	204.006	5.649
90	1, $R_{sp,t-1}$	Continuous	Annual	31.595	7.379	11.756	27.709	5.350	337.319	29.997	2.216	15.984
350	1, $R_{sp,t-1}$	2-Step	Annual	29.652	11.354	4.625	3.777	1.059	6.854	36.434	4.820	31.815
350	1, $R_{sp,t-1}$	2-Step	Monthly	23.915	34.859	29.856	1.963	0.686	0.844	32.300	45.575	39.018
350	1, $R_{sp,t-1}$	Iterative	Monthly	1.462	30.578	88.591	0.591	34.073	187.088	-13.334	29.124	292.519
90	1, $R_{sp,t-1}$	2-Step	Annual (ρ=40)	33.011	143.054	15.729	42.392	158.339	40.899	1.073	0.143	0.625
90	1, $R_{sp,t-1}$	2-Step	$Corr(C_t,C_{t-1}) = -0.30$	30.824	19.512	11.323	3.805	8.824	5.678	32.419	6.119	30.565
90	1, $R_{sp,t-1}$	2-Step	$Corr(C_t,C_{t-1}) = -0.50$	33.199	6.918	30.082	2.424	0.337	2.600	29.431	3.865	47.101
5000	1, $R_{sp,t-1}$	2-Step	Annual	29.137	3.745	0.832	2.006	0.071	0.072	28.082	1.909	1.840
5000	1, $R_{sp,t-1}$	Iterative	Annual	29.169	3.616	1.218	2.016	0.078	0.294	29.768	1.701	9.004

Table 7
Robustness of Monte Carlo Simulation Results to Changes in Sample Size, DGP, Instrument Set and Weighting Matrix Using "Extreme" Parameter Value α = 29 and ρ = 2 (unless specified otherwise).

Panel B: Proportion of rejections of goodness-of-fit $J_{T,EZ}$ test and power of hypothesis tests

Sample Size	Instrument Set	Weighting Matrix	Endowment Process	Proportion of Rejections at nominal size (0.05)	Size-Adjusted Power of Tests of TSEU Null Against EZ (empirical size = 0.05): D	Wald	$J_{T,TSEU}$
90	1, $R_{sp,t-1}$	2-Step	Annual	0.086	0.040	0.430	0.060
90	1	2-Step	Annual	-	0.125	0.010	0.050
90	Optimal	2-Step	Annual	-	0.065	0.685	0.065
90	1, $R_{sp,t-1}$	Continuous	Annual	0.270	-	0.785	0.620
350	1, $R_{sp,t-1}$	2-Step	Annual	0.058	0.020	0.376	0.106
350	1, $R_{sp,t-1}$	2-Step	Monthly	0.018	0.010	0.332	0.172
350	1, $R_{sp,t-1}$	Iterative	Monthly	0.216	0.008	0.384	0.264
90	1, $R_{sp,t-1}$	2-Step	Annual (ρ=40)	0.240	0.075	0.105	0.115
90	1, $R_{sp,t-1}$	2-Step	Corr(C_t,C_{t-1}) = -0.30	0.030	0.005	0.120	0.095
90	1, $R_{sp,t-1}$	2-Step	Corr(C_t,C_{t-1}) = -0.50	0.065	0.025	0.360	0.625
5000	1, $R_{sp,t-1}$	2-Step	Annual	0.005	1.000	0.950	0.995

5000	1, $R_{sp,t-1}$	Iterative	Annual	0.200	1.000	0.800	1.000

Table 8

Condition Indexes and Variance Decomposition Proportions. Mean Values Across

500 Draws of 90 Observations from the Annual DGP

Panel A: $\alpha = 0.80$, $\rho = 0.80$, Annual DGP

		Proportions of	
Condition Index	Var(β)	Var(ρ)	Var(γ)
1.000	0.014	0.002	0.040
6.968	0.057	0.016	0.414
79.612	0.931	0.982	0.545

Panel B: $\alpha = 29.0$, $\rho = 29.0$, Annual DGP

		Proportions of	
Condition Index	Var(β)	Var(ρ)	Var(γ)
1.000	0.095	0.164	0.010
1.914	0.168	0.375	0.171
7.685	0.737	0.478	0.729

Panel C: $\alpha = 29.0$, $\rho = 2.0$, Annual DGP

		Proportions of	
Condition Index	Var(β)	Var(ρ)	Var(γ)
1.000	0.054	0.101	0.054

11.625	0.119	0.240	0.120
238.687	0.826	0.659	0.827

Panel D: $\alpha = 29.0$, $\rho = 29.0$, $Corr(C_t, C_{t-1}) = -0.50$

		Proportions of	
Condition Index	<u>Var(β)</u>	<u>Var(ρ)</u>	<u>Var(γ)</u>
1.000	0.163	0.282	0.167
1.311	0.272	0.323	0.289
2.058	0.565	0.395	0.545

Table 8 (Continued)

Condition Indexes and Variance Decomposition Proportions. Mean Values Across

500 Draws of 90 Observations from the Annual DGP

Panel D: $\alpha = 29.0$, $\rho = 2.0$, $Corr(C_t, C_{t-1}) = -0.50$

		Proportions of	
Condition Index	Var(β)	Var(ρ)	Var(γ)
1.000	0.037	0.185	0.042
2.781	0.133	0.291	0.104
39.922	0.830	0.524	0.854