

# A Theory of the Transition to Secondary Market Trading of IPOs

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## **Abstract**

We developed a model in which investment banks and institutional investors collaborate in smoothing an IPO's transition to secondary market trading. Their intervention promotes welfare under the assumption that significant new information arrives in the market in the immediate aftermath of the IPO. Under this assumption, it is optimal to stage the offering and suboptimal to commit to selling shares at a uniform price. The optimal strategy yields an economic rationale for secondary market price stabilization for IPOs carried out via a well-coordinated network of repeat institutional investors.

# 1 Introduction

Initial public offerings of equity (IPOs) constitute the creation of a new market. Following an extended effort to gauge demand for the issuing firm's shares, an initial price is set, shares are placed at this price and a secondary market is thrown open for trade. Despite the best efforts of the issuer's investment bank, the degree of informational asymmetry within the investor community often remains large as evidenced by massive share turnover and price volatility. In the extreme, such informational friction can lead to market failure. Thus a potentially significant but temporary need for intermediation persists during the transition to normal secondary market trading conditions.

The investment bank's primary functions during this transition include the initial distribution of shares, predominantly among institutional participants in their marketing effort, and a variety of practices aimed at "stabilizing" the secondary market price.<sup>1</sup> In this paper we model the interplay between share distribution and price stabilization functions during the transition to secondary market trading. We use the term "transition" precisely to emphasize the *process* by which new issues begin "normal" secondary market trading. Our model envisions initial allocations reflecting not an attempt to place shares in the hands of the ultimate investors but rather as part of a process of distributing shares optimally in light of extraordinary incremental information arrival triggered by the IPO.

The existing literature studies how such information is incorporated in the offering price via a bookbuilding effort. Our contribution is to suggest that the exclusivity and narrow time window within which this effort is carried out precludes subsequent, potentially equally important, information finding its way into the market until after the IPO is formally completed. The issuing firm, recognizing that its actions trigger incremental information arrival beyond that revealed in the bookbuilding effort, might prefer to sell its shares incrementally and, perhaps, at a non uniform price to extract more surplus from informed investors. The issuing firm is bound, however, both by regulatory constraints and timing considerations.

In this setting concentrating initial allocations in the hands of institutional investors whose secondary market trading behavior can be influenced by the investment bank enables the issuer to increase its expected

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<sup>1</sup>The SEC defines price stabilization as "... transactions for the purpose of preventing or retarding a decline in the market price of a security to facilitate an offering." Such practices are permitted because: "Although stabilization is a price-influencing activity intended to induce others to purchase the offered security, when appropriately regulated it is an effective mechanism for fostering an orderly distribution of securities and promotes the interests of shareholders, underwriters, and issuers." (SEC release No. 34-38067, p.81, Dec. 20, 1996. This release announces the adoption of Regulation M to replace Exchange Act rules 10b-6, 10b-6A, 10b-7, 10b-8, and 10b-21 (the trading practice rules).

revenues by blurring the distinction between primary and secondary markets. We demonstrate that the optimal pricing and allocation mechanism has features consistent with existing distribution and price stabilization practices.

To the best of our knowledge, this is the first theory that explicitly ties price stabilization to share distribution in the positive spirit in which it is permitted by the SEC.<sup>2</sup> Thus our analysis complements a rapidly growing body of empirical evidence describing distribution and price stabilization practices for IPOs.<sup>3</sup> It also sheds light on the recent regulatory debate surrounding proposals to amend Regulation M in response to practices that gained notoriety during the dot-com era.<sup>4</sup>

We model the arrival of incremental information in the early stages of secondary market trading by assuming that investors arrive exogenously in two "waves."<sup>5</sup> Each investor observes either a high value signal or a low value signal. The signals observed by early arrivals are "durable" in the sense that the true secondary market value reflects the collection of signals arriving with each wave of investors. In other words, the secondary market value of the issuer's shares reflects a common value information structure.

We demonstrate in this setting that the issuer optimizes by "staging" the sale of IPO shares into the secondary market rather than selling the entire offering immediately at a uniform price. The intuition for the staged sale rests with its capacity for promoting competition between early and late arrivals and thus has the flavor of intertemporal price discrimination mechanisms favored by monopolists in durable goods [e.g., Tirole (1988)]. Early arrivals pay a higher price than late arrivals in equilibrium under the threat of being crowded out if they delay their share purchases. Thus our model predicts secondary market price dynamics consistent with commonly observed sharp initial price increases followed by price declines.

In practice, issuers do not sell shares directly to investors and IPO shares must be sold at a uniform price. Our analysis demonstrates that the uniform-price constraint can be neutralized by price stabilization practices that control secondary market sales by institutional investors receiving initial allocations. An issuer who deals infrequently with institutional investors will be incapable of exercising this control. Thus

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<sup>2</sup>See Benveniste, Busaba, and Wilhelm (1996) and Chowdhry and Nanda (1996) for alternative perspectives.

<sup>3</sup>See Aggarwal (2000, 2003), Aggarwal and Conroy (2000), Aggarwal, Prabhala, and Puri (2001), Benveniste, Erdal and Wilhelm (1998), Boehmer and Fishe (2000, 2002), Boehmer, Boehmer and Fishe (2004), Cornelli and Goldreich (2001, 2003), Fishe (2001), Ellis, Michaely and O'Hara (2000), Fishe (2001), Griffin, Harris and Topaloglu (2004), Hanley and Wilhelm (1995), Hanley, Kumar, and Seguin (1993), Hanley, Lee, and Seguin (1996), Jenkinson and Jones (2004), Krigman, Shaw and Womack (1999) and Ljungqvist and Wilhelm (2002).

<sup>4</sup>See the SEC's proposed amendments to Regulation M at <http://www.sec.gov/news/press/2004-145.htm>.

<sup>5</sup>The model can be generalized to allow costly information production but at considerable complication to the issuer's objective function. Sequential information arrival in our model is intended to abstract from the more complicated setting while maintaining its spirit – that all potential investors do not obtain information simultaneously.

issuers engage investment banks to gain access to institutional investor networks within which the bank can influence secondary market trading behavior. Influence over institutional investors derives from the power to exclude them from future IPOs, essentially an informal "penalty bid." We demonstrate in a dynamic setting that the offer price and initial secondary market trading prices can thus be determined jointly in equilibrium such that institutional investors cooperate in achieving the optimal staging and the issuer's expected revenue is equivalent to that achieved via the optimal direct sale.

In essence, we argue that the well-documented tendency to place a large fraction of IPO shares with institutional investors [see Hanley and Wilhelm (1995), Ljungqvist and Wilhelm (2002) and Aggarwal (2000, 2002)] reflects an intermediate stage in the final distribution of an IPO. With this initial distribution in place, the uniform-price constraint is satisfied. Price stabilization practices aimed at controlling the sale of initial allocations [see Benveniste, Busaba, and Wilhelm (1996) and Aggarwal (2000, 2003)] then enable the investment bank to respond optimally, on behalf of the issuer, to the arrival of informed secondary market investors.

Proposed amendments to Regulation M suggest prohibiting formal penalty bids. These contractual arrangements call for syndicate members to forfeit selling concessions when their investors immediately "flip" initial allocations in the secondary market. Our analysis suggests that penalties of this sort are too weak to enforce cooperation among institutional investors. Their proposed prohibition reflects concerns for their opacity and a perceived burden on retail investors, against whom it is believed they are most frequently exercised. We do not provide a countervailing economic argument in favor of such formal penalty bids especially when exercised against retail investors.<sup>6</sup>

Our analysis rests on several key assumptions. Moreover, it begs questions regarding recent calls for and experiments with auctioning IPOs.<sup>7</sup> We examine both the robustness of our conclusions and their relation to the debate surrounding the desirability of replacing traditional practice with auctions mechanisms by recasting the model as an English auction carried out in a private values setting. Bulow and Klemperer (1996) derive an important result in this setting. Namely, if there is no cost to doing so, it is always desirable to delay a sale to include more participants in a private value, second-price auction. In our setting, this is tantamount to saying that the issuer should simply wait out the arrival of the two waves of

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<sup>6</sup>Benveniste, Busaba, and Wilhelm (1996) suggest that penalty bids exercised against retail investors might promote efficiency in the bookbuilding effort.

<sup>7</sup>See Wilhelm (2005) for an overview.

information signals and then conduct an auction. We replicate this result for the information structure at hand.

We contend, however, that the IPO presents a setting in which delay costs can be consequential. Moreover, although we do not model incentives for information production, we believe that the production of the incremental information that arrives in two waves in our model is best thought of as being triggered by the initial pricing and placement of shares. Rationally or not, issuers often perceive a "window of opportunity" for their IPO related to broader market conditions, industry conditions or investment opportunities.<sup>8</sup> Alternatively, until the firm goes public it is captive to presumably more costly sources of finance [see Schenone (2005)]. Thus we examine the case where the auction can be staged and delayed sale is costly. In this setting, we characterize the magnitude of delay costs that merit staging the auction and show that when the auction is staged, the price dynamics derived in the original model are preserved.

Aggarwal, Purnanandam and Wu (2005) derive price dynamics like ours (positive returns followed by price reversals) from tie-in agreements requiring additional share purchases in the secondary market in exchange for initial allocations.<sup>9</sup> This practice, apparently common during the dot-com era, is referred to as "laddering." Our approaches are distinguished by assumptions regarding where, within the investor community, informational advantage resides. Aggarwal et al. assume that bank "affiliated" institutional investors have information about the future share value that is unknown to secondary market investors.<sup>10</sup> By contrast, we implicitly assume that the (uniform) offer price reflects this information. Thus, coupled with the fact that they receive incremental information signals, secondary market investors maintain the advantage in our model.

## 2 The Model

The economy comprises issuing firms and three types of risk-neutral agents: an intermediary (investment bank), institutional investors and secondary market investors. We analyze two selling mechanisms. The

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<sup>8</sup>A noteworthy example is Andover.net's December 7, 1999 IPO auction carried out by W.R. Hambrecht. When the auction closed, the clearing price stood at \$24 but Andover.net elected to sell the IPO within the suggested price at \$18 per share. Offering the shares at \$24 would have imposed a delay associated with refiling the price with SEC. VA Linux, a close competitor, planned to go public the following day. See *The Wall Street Journal*, December 9, 1999, p. C1.

<sup>9</sup>Ljungqvist, Nanda and Singh (2005) obtain the same price dynamics in expectation but under the assumption that some investors are irrational. Our results arise in expectation in a fully rational model.

<sup>10</sup>Laddering is socially costly but profitable for the investment bank and affiliated institutional investors in a pooling equilibrium that is more readily sustained during periods of "high investor sentiment" when more secondary market investors are inferring from (manipulated) prices and volumes whether to buy shares themselves.

first abstracts from any need for an intermediary and thus both the practice of building a book for the offering prior to setting the offer price and any conflicts of interest between the issuer and its intermediary [e.g., Baron (1982), Baron and Holmstrom (1980), Biais et al (2002)]. In this context, issuing firms directly sell  $n$  shares of stock in a public offering with the objective of maximizing expected revenue from the sale. The shares are purchased by terminal or “secondary market” investors who observe an informative signal before being approached by the issuer. Secondary market investors are not easily identified by the issuer and thus we assume that they cannot be approached simultaneously. Alternatively, one can think of the offering being impossible to place instantaneously.

For simplicity we assume that shares are distributed in two stages and that secondary market investors at each stage observe new information regarding the firm prior to being approached by the issuer. Thus our modeling approach contrasts with that in the literature stemming from Benveniste and Spindt (1989) in the sense that we explicitly assume that information bearing on the issuer’s valuation continues to arrive after the bookbuilding effort for the IPO is completed. Under this assumption, we derive an optimal strategy for staging the offering that calls for different prices for shares placed at different times and serves as a benchmark for subsequent analysis.

In practice, U.S. securities regulations require that all shares in an offering be sold at a uniform price. In the second mechanism we introduce an intermediary, the investment bank, with whom the issuer contracts to place the entire offering at a uniform price.<sup>11</sup> For simplicity, we assume that the entire offering is placed initially with institutional investors whose secondary market trading can be influenced by the investment bank. The investment bank adds value to the extent that it influences institutional investors to sell their shares in the secondary market according to the two-stage benchmark strategy. Institutional investors are meant to act as members of a distribution team for the offering but their private interests conflict with the collective interest in the optimally staged benchmark strategy. If the investment bank can resolve this coordination problem, the initial distribution to institutional investors provides a synthetic mechanism for non-uniform pricing. But even in the absence of regulations requiring uniform pricing, having institutional investors participate in this staged distribution of shares to "final" shareholders is

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<sup>11</sup>Once again, we abstract from the bookbuilding process as well as reputational concerns [e.g., Chemmanur and Fulghieri (1994)] that motivate the presence of an intermediary and assume that the interests of the issuer and the intermediary are aligned with one another.



secondary market investor's demand is for one share. Secondary market investors cannot sell short.<sup>12</sup> Finally, we assume that

$$m_1 < n < m_1 + m_2. \tag{1}$$

This assumption motivates staged placement of shares with the ultimate secondary market investors and ensures the existence of both date 1 and date 2 prices. The economic rationale for the assumption lies in physical limits to instantaneous distribution at one point in time and expectations regarding the arrival of subsequent information that provide incentive for secondary market investors to delay bidding for shares. In developed markets, initial allocations can take place within a matter of hours or even minutes but we envision some, if not all, initial allocations to institutional investors exceeding their optimal allocations. As such, these investors are best viewed as members of the distribution team and thus the time between date 1 and date 2 can be thought of as corresponding to the time required for them to dispose of shares in excess of their optimal allocation.

## 2.1 Information Structure

Just prior to date 1, each secondary market investor  $i$  among the  $m_1$  date 1 investors observes a signal,  $s_i$ , that with equal probability takes either a high value ( $s_i = h$ ) or a low value ( $s_i = l$ ) where  $h > l$ . A second round of signals is observed by each of the  $m_2$  secondary market investors just prior to date 2. All signals  $s_i$  and  $s_j$  are independent for  $i \neq j$ . The exogenous incremental arrival of investors and signals reflects the intuition that early secondary market uncertainty provides incentive for further information production.

The secondary market value of each share of stock is a function of the investor signals taking the form

$$V(s_1, s_2 \dots s_{m_1+m_2}) = s_1 + s_2 + \dots s_{m_1+m_2}. \tag{2}$$

Thus we assume a common value information structure given the signals received by secondary market investors and, for the sake of tractability, that the value function is linear. These assumptions are standard

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<sup>12</sup>Geczy et al. (2002, p.266) provide evidence that "short exposure to IPOs is generally feasible for those with good access to equity loans, even in the first days of trading" but initially expensive with supply constraints during the first 30 days of trading arising from brokerage rules forbidding purchases on margin and agreements among syndicate members to not lend shares. Duffie et al. (2002) and D'Avolio (2002) provide theory and evidence of "endogenous support" (D'Avolio, p.302) for the assumption of constrained short selling on the grounds that the costs of short selling are systematically high when investors disagree most as one might expect during early secondary market trading.

in the market microstructure literature stemming from Kyle (1986) and they set aside efficiency concerns arising in a private values model related to whether shares are placed with those who value them most.<sup>13</sup>

### 3 A Benchmark Equilibrium

In this section we establish the benchmark equilibrium for direct placement of shares with secondary market investors in the absence of a uniform-price constraint.

#### 3.1 Sequence of Events

The entire offering cannot be sold at date 1 ( $m_1 < n$ ) but must be sold by date 2. The issuer sets a price at each date,  $p_1$  and  $p_2$ , to maximize total revenue from the staged offering. At date 1, each of the  $m_1$  secondary market investors decides, conditional only on his own signal whether to bid for one share at the per share price  $p_1$ . Public information at date 1,  $\varphi_1$ , is the empty set. The number of date 1 bidders,  $d_1$  becomes public information after bidding takes place. By construction, each date 1 bidder receives a share at price  $p_1$ .

At date 2, date 1 investors who did not buy shares at date 1 and all date 2 investors, conditional on their own signal and public information  $\varphi_2 = \{p_1, d_1\}$ , decide whether to bid at the new price  $p_2$ . If there are fewer bidders than shares available, then each bidder receives a share at price  $p_2$ . If there are more bidders than shares available, the available shares are rationed among the bidders with equal probability.

#### 3.2 Definition of Equilibrium

The conditions for a Perfect Bayesian Equilibrium (*PBE*) are provided in definition 1.

**Definition 1** *An equilibrium is composed of the following elements:*

*i) The issuer's pricing rule,  $p_t(\varphi_t)$ ,  $t = 1, 2$ ;*

*ii) Investor  $i$ 's bidding strategy as reflected in the probability of bidding at each date,  $b_i^j(p_t, \varphi_t, s_i)$ ,*

*such that at each date, a)  $p_t$  maximizes the issuer's continuation payoff given investors' bidding strategies and his belief regarding their private information,  $\mu_t^s \equiv \{\pi_1^s(t) \dots \pi_{m_1+m_2}^s(t)\}$ , where  $\pi_i^s(t) = \Pr\{s_i =$*

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<sup>13</sup>One might justify a private values model on the grounds that some investors face liquidity shocks or different hedging demands. We explore this alternative later but note that the intuition from the common values setting is preserved under the assumption of private values.

$h|\varphi_t$ ; b)  $b_t^i$  maximizes investor  $i$ 's continuation payoff given the bidding strategies of others and his beliefs regarding their private information,  $\mu_i^t \equiv \{\pi_1^i(t) \dots \pi_{i-1}^i(t), \pi_{i+1}^i(t), \dots, \pi_{m_1+m_2}^i(t)\}$ , where  $\pi_j^i(t) = \Pr\{s_j = h|\varphi_t, s_i\}$ ; c) beliefs are consistent with the equilibrium strategies of the issuer and investors; d) the sale of shares is completed at date 2.

The issuer is permitted a mixed strategy in the sense that  $p_t(\varphi_t)$  is a probability distribution function on the real line. However, we focus on symmetric equilibria where all the date 1 investors follow the same bidding strategy as do all date 2 investors. We demonstrate later that focusing on symmetric equilibria is without loss of generality because optimal equilibria are symmetric along the equilibrium path. Optimal equilibria provide the issuer with the highest *ex ante* expected payoff among all equilibria.

### 3.3 Equilibrium Analysis

In equilibrium, the issuer maximizes his expected payoff by setting date 1 and date 2 prices to induce potential bidders receiving different signals to bid at different prices and dates. Using backward induction, we first examine equilibrium strategies for date 2 agents taking date 1 strategies as given and then show that agents' date 1 strategies constitute an equilibrium. Denoting  $d_1^{-i}$  as the number of date 1 bidders except investor  $i$ , equilibrium is summarized in the following proposition:

**Proposition 2** *There exists a symmetric equilibrium in which*

- (i) *Date 1 investors who receive  $h$  signals bid with probability  $b^*$ , where  $b^*$  is determined by equation (13) in the appendix; Date 1 investors who receive  $l$  signals do not bid at date 1.*
- (ii) *Date 1 investors who did not bid at date 1 and all date 2 investors bid at date 2 and shares not sold at date 1 are rationed among them.*
- (iii) *the issuer sets  $p_1$  and  $p_2$  as follows*

$$\begin{aligned} p_2^* &= E[V|s_j = l, d_1], \quad \forall j \in I_2, \\ p_1^* &= E[V|s_i = h] - \alpha(h-l) \frac{2 - \frac{1}{2}b^*}{2 - b^*} E\left[\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}\right], \quad \forall i \in I_1, \end{aligned}$$

where expectations are taken given investors' equilibrium strategies.

- (iv) *Along the equilibrium path  $p_1^* \geq E[p_2^*]$ .*
- (v) *if  $m_2 \geq 3n - m_1$ ,  $p_1^* \geq E[V]$ .*

Condition (iv) implies that the issuer and investors expect the selling price to decline from date 1 to date 2 along the equilibrium path. Condition (v) implies that date 1 share sales will occur at a price exceeding their expected value. In sum, the necessity of staging the sale of shares across time coupled with the arrival of new information at each point in time is sufficient to yield initial share prices exceeding their expected value in equilibrium and declining prices for shares sold later in the distribution.

Although they expect the share price to decline, date 1 investors receiving  $h$  signals prefer to buy shares at date 1 for fear of being crowded out by a large number of date 2 investors. Specifically, for an investor  $i$  receiving an  $h$  signal at date 1, bidding at price  $p_1$  ensures a share allocation with expected payoff  $E[V|s_i = h] - p_1$ . If the same investor delays bidding until date 2, he expects to face a lower price but may not receive a share allocation. Conditional on receiving an allocation his payoff is  $E[V|s_i = h, d_1^{-i}] - p_2(d_1^{-i})$ . But in equilibrium the issuer sets prices such that the date 1 investor receiving an  $h$  signal is indifferent between bidding at date 1 and date 2 or to just satisfy investor  $i$ 's incentive compatibility condition:

$$E[V|s_i = h] - p_1^* = E\left\{\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}[E[V|s_i = h, d_1^{-i}] - p_2^*(d_1 = d_1^{-i})]\right\}, \quad (3)$$

where  $\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}$  is the probability that investor  $i$  receives a share allocation at date 2 conditional on  $d_1^{-i}$ . We show in the appendix that  $E[V|s_i = h, d_1^{-i}] - p_2^*(d_1 = d_1^{-i})$  does not depend on  $d_1^{-i}$ . Therefore, (3) can be rewritten as

$$\begin{aligned} E[V|s_i = h] - p_1^* &= E\left[\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}\right]E[V|s_i = h, d_1^{-i}] - p_2^*(d_1 = d_1^{-i}) \\ &= E\left[\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}\right]\{E[V|s_i = h] - E[p_2^*(d_1 = d_1^{-i})]\}. \end{aligned}$$

Since  $E\left[\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}\right] < 1$ , it follows immediately that  $p_1^* > E[p_2^*(d_1 = d_1^{-i})]$ . The date 1 investor's probability of receiving a share allocation if he delays bidding until date 2,  $E\left[\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}\right]$ , is decreasing in the expected number of date 2 investors,  $m_2$ . When  $m_2$  is large, date 1 investors receiving high signals pay a price that approaches the expected value conditional on their signal  $E[V|s_i = h]$ . This implies a date 1 price that exceeds the stock's unconditional expected value,  $E[V]$ . Thus, in equilibrium, the threat of crowding out by date 2 investors provides incentive for date 1 investors with positive information to bid early and aggressively.

The seemingly paradoxical potential for  $p_1^* > E[V]$ , does not imply that date 1 investors expect to lose money on their investment. Recognize first that  $E[V|d_1]$  is increasing in  $d_1$  and thus  $E[V|d_1]$  and  $d_1$  are positively correlated because only investors with high signals bid at date 1. Thus the date 1 investor's expected profit conditional on his high signal is

$$\begin{aligned} E\{[E[V|d_1] - p_1^*]d_1\} &= E\{E[V|d_1] - p_1^*\}E[d_1] + cov[\{E[V|d_1] - p_1^*\}, d_1] \\ &= \{E[V] - p_1^*\}E[d_1] + cov[\{E[V|d_1] - p_1^*\}, d_1]. \end{aligned} \quad (4)$$

In equilibrium, date 1 demand for the stock contains information regarding the issuer's share value. Even when  $E[V] - p_1^* < 0$ , a sufficiently large covariance between the number of date 1 bidders and the (conditional) expected share value, yields positive expected profits for date 1 investors.<sup>14</sup>

Moreover, in equilibrium more aggressive date 1 bidding yields a higher date 2 price:

**Corollary 3** *The date 2 price,  $p_2^*$ , is increasing in  $d_1$ , the date 1 demand for shares.*

Intuitively, when share value is determined by investor signals arriving incrementally and immediate (date 1) demand (or distribution capacity) does not exhaust the number of shares for sale, early investors (those receiving positive signals) benefit from wider dissemination of the positive signal: the more shares they buy, the larger their per share expected profit.<sup>15</sup>

The preceding analysis focuses on one symmetric equilibrium. The following proposition demonstrates that focusing on symmetric equilibria does not sacrifice generality if our interest is confined to "optimal" equilibria that maximize the issuer's expected revenue.

**Proposition 4** *The optimal equilibrium path is characterized by Proposition 2.*

Optimal equilibria differ only off the equilibrium path. Thus the pricing scheme derived in 2 is a general prediction of the model.

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<sup>14</sup>The covariance between  $E[V|d_1]$  and  $d_1$  reflects the sensitivity of expectations to bidding behavior. We conjecture that this sensitivity increases with the precision of the private information reflected in bids. In our model the precision of private information increases with the spread ( $h - l$ ) between high and low signal values.

<sup>15</sup>Although we do not permit short selling, it would be a profitable strategy for a date 1 investor when  $p_1^* > E[p_2^*]$  and would destroy the equilibrium assuming that the investment bank (acting as market maker) purchases shares at  $p_1^*$ . Thus a capacity for short selling decreases the issuer's expected net proceeds.

## 4 An Intermediated Mechanism

### 4.1 The Coordination Problem

Under the assumption that all shares cannot be sold simultaneously and that new information arrives during the time required for their complete distribution, the benchmark revenue-maximizing equilibrium for the direct sale of IPO shares to terminal investors yields two insights:

- Early recipients of information that bears positively on share value will bid early and aggressively under the threat that failing to do so risks being crowded out by later recipients of new information.
- The price paid by early bidders optimally exceeds that paid by later bidders.

The benchmark equilibrium assumes, unrealistically (at least from an historical perspective), that the issuer approaches potential terminal investors directly. Introducing an intermediary such as an investment bank addresses this issue but raises a practical problem: U.S. securities regulations prevent non-uniform pricing of shares in a securities offering. As a consequence, the investment bank is prohibited from direct execution of the revenue maximizing staged sale of IPO shares.

Even if regulation did not require uniform pricing, with costly risk capital, it may be suboptimal for a single bank to bear the underwriting burden of the staged offering. In fact, the origins of modern underwriting syndicates lie in the rapidly increasing scale of late 19th century securities offerings coupled with months long distribution efforts necessitated by primitive transportation and communications technology.

The problem with collective execution of the staged offering stems from the declining price schedule that arises in equilibrium. Any single member of the distribution team recognizing that shares prices will decline over time has incentive to sell shares sooner rather than later. But if everyone tries to sell at date 1, the equilibrium price  $p_1^*$  cannot be sustained. Of course the syndicate manager can resolve the problem by offering to buy back excess shares for  $p_1^*$ , effectively serving as a market maker in the secondary market, but that simply shifts the underwriting burden back to a single party, presumably the syndicate manager. If the manager is capital constrained, the only implementable equilibria are those in which  $p_1 \leq E[p_2]$ . Such equilibria are suboptimal and thus imply suboptimal financing terms for the issuer. Thus even if we ignore the uniform-price constraint, the optimal staged offering presents a serious coordination problem.

The solution we propose to this joint problem recognizes that once the syndicate allocates shares at a uniform price to initial investors, subsequent sales of those initial allocations can take place at any price the secondary market will bear. Institutional investors are the primary recipients of these initial allocations [see Hanley and Wilhelm (1995), Ljungqvist and Wilhelm (2002) and Aggarwal (2003)]. Although institutional investors have the same incentives to defect from the optimal staged equilibrium by attempting to “flip” their initial allocations as quickly as possible, they are distinguished from other investors by virtue of their repeated dealings with the investment bank. In the next section we examine how the investment bank can use leverage gained through their repeated dealings to indirectly execute the optimal staged offering through its institutional investor network.

## 4.2 Resolving the Coordination Problem: A Dynamic Model

We extend the model to include investors and an investment bank and abstract from the transfer of shares between the issuer and the bank by simply assuming that their interests are perfectly aligned. Both institutional investors and the investment bank maximize their private discounted payoffs.

We represent the bank’s relationship with institutional investors as an infinite repeated game. In each period, the investment bank brings one IPO to market. Secondary market investors are short-sighted in the sense that they live for only one period.<sup>16</sup>

In each period, the bank selects  $k$  institutional investors with whom it wishes to place the entire offering at a uniform offer price  $p_0$ . We assume an abundance of institutional investors so that if a chosen investor decides not to participate the bank simply invites another institutional investor to join the offering. The payoff to an institutional investor not participating in the offering is standardized to be 0. The  $k$  participating institutional investors receive the same allocation. Discriminatory allocation complicates the analysis but does not change our conclusions.

When the initial allocation is completed, secondary market trading begins. Each institutional investor has two options. First, he could sell his share allocation according to a plan established by the investment bank. If this option is taken, the investment bank effectively decides whether the investor’s shares are

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<sup>16</sup>We make this additional assumption for the purpose of distinguishing secondary market investors from institutional investors with respect to their potential for entering a strategic relationship with the investment bank. Alternatively, we might argue that the pool of secondary market investors is not stable and thus it is unlikely that the investment banker could influence their behavior as it might that of institutional investors with which it deals repeatedly.

sold at date 1 or date 2. Alternatively, the institutional investor can flip his allocation by selling it in its entirety at date 1 for a per share price of  $p_1$ .<sup>17</sup>

Without loss of generality, we assume that institutional investors make decisions independent of any information regarding date 1 demand  $d_1$  and that shares sold at date 1 are repurchased by the investment bank at the date 1 price  $p_1$ . The latter assumption implies that in its market making capacity, the investment bank maintains a zero bid-ask spread. As before, short selling is not permitted.

Each IPO (or period) in the infinite series plays out in this manner and both the bank and institutional investors have a cross-period discount factor  $\delta < 1$ . Thus a large discount factor implies that institutional investors place a high premium on future cash flows or, alternatively, are heavily dependent on their relationship with the bank. Finally, we assume that the bank observes the trading behavior of institutional investors. This assumption is consistent with modern technology that enables ex post monitoring of the movement of IPO shares.

We analyze the model in the framework of Abreu, Pearce and Stacchetti (1990). The following proposition establishes the conditions and characterizes the *perfect public equilibrium* (PPE) under which the optimal staged offering can be implemented indirectly by an intermediary through a pool of institutional investors with whom it deals repeatedly:

**Proposition 5** For  $\delta > \frac{p_1^* - \frac{E[V]n - S}{n}}{E[V] - \frac{S}{n} + p_1^* - \frac{E[V]n - S}{n}}$ , there is a PPE in which

(i) Institutional investors are allocated IPO shares at a uniform offer price

$$p_0^* = \left(E[V] - \frac{S}{n}\right) - \frac{1 - \delta}{\delta} \left(p_1^* - \frac{E[V]n - S}{n}\right),$$

where  $S$ , as defined in the appendix, is the total surplus captured by secondary market investors by virtue of information signals received at date 1 and date 2.

(ii) In every period, institutional investors sell their shares according to the benchmark strategy described in proposition 2.

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<sup>17</sup>Permitting the institutional investor to sell a fraction of the initial allocation at a particular date does not change our conclusions. We could also provide institutional investors with the option to hold their shares until date 2 in the secondary market. However, this option can be omitted without loss of generality if our interest lies in implementing the optimally staged sale because in any equilibrium where institutional investors have no incentive to flip, the same incentive can be used to prevent their suboptimally holding their shares until date 2.

(iii) Institutional investors deviate from the optimal strategy at the risk of exclusion from all future IPOs.

(iv) Institutional investors collectively expect a per period payoff,  $n\frac{1-\delta}{\delta}(p_1^* - \frac{E[V]n-S}{n}) > 0$ , therefore all institutional investors chosen to participate agree to do so.

(v) The offer price  $p_0^*$  is increasing in the discount factor  $\delta$ .

(vi) The equilibrium provides the investment bank with the highest payoff among equilibria that implement the optimal staged sale described in proposition 2.

Proposition 5 reveals that the coordination problem associated with staged distribution can be resolved when institutional investors place sufficient value on their relationship with the investment bank. Institutional investors are rewarded for cooperative participation in the distribution of IPO shares to secondary market investors by regular opportunities to obtain allocations at the discounted, but uniform, offer price. Point (i) in 5 demonstrates that the offer price is discounted from the unconditional expected share value to compensate institutional investors both for expected losses to better informed secondary market investors and for not flipping shares at the date 1 price,  $p_1^*$ , where,  $E[V] - \frac{S}{n}$  is the equilibrium expected selling price for each institutional investor. From Proposition 2,  $p_1^* > E[p_2^*]$ , we know that  $E[V] - \frac{S}{n}$  is smaller than  $p_1^*$  because in equilibrium there is a positive probability that an institutional investor's share will be sold at a lower price,  $p_2^*$ . As the discount factor,  $\delta$ , increases, the bank effectively has more leverage with institutional investors and thus is able set the offer price more aggressively while still gaining their cooperation in the distribution of the offering [point (v)]. Thus banks that maintain stronger relationships with a stable pool of institutional investors provide issuers with larger expected revenues from IPOs. If bank prestige corresponds with the quality of the bank's investor network, then more reputable banks will underprice IPOs less for the purpose of achieving optimal distribution.

Understanding point (vi) requires closer examination of the institutional investor's incentive to flip an initial allocation. Deviation from the optimal staging of the IPO distribution yields the institutional investor an immediate payoff of  $(p_1^* - \frac{E[V]n-S}{n})\frac{n}{k}$  but at the cost of exclusion from future IPOs. Thus the institutional investor's incentive compatibility constraint is

$$\sum_{\tau=1}^{\infty} \delta^{\tau} \frac{n}{k} (E[V] - \frac{S}{n} - p_0) \geq (p_1^* - \frac{E[V]n-S}{n}) \frac{n}{k}. \quad (5)$$

Since  $E[V] - \frac{S}{n}$  is the per share joint surplus of the investment bank and institutional investor,  $\frac{n}{k}(E[V] - \frac{S}{n} - p_0)$  is the institutional investor's surplus from participating in the distribution of a single IPO. In equilibrium the investment bank's payoff (and that of the issuer) is maximized when the constraint is binding. The investment bank's cost of sustaining cooperation is minimized by permanent exclusion of institutional investors who deviate from the optimal distribution strategy. Anything short of this threat increases the discount on initial allocations necessary to sustain cooperation and thus increases observed underpricing and reduces the issuers expected revenue from the offering.

The severity of the equilibrium punishment threat may shed light on the apparent laxity with which formal penalty bids are enforced. Formal penalty bids are contractual agreements among underwriting syndicate members that call for members to forfeit selling concessions when shares they've placed with initial investors are flipped in the secondary market. Presumably, the threat of forfeiting the selling concession leads members of the distribution team to discourage investors with whom they place shares from flipping their allocations. Aggarawal (2003) observes that penalty bid enforcement is rare. Our analysis suggests that *formal* penalty bids do not have sufficient force to sustain cooperation among institutional investors who receive allocations in exchange for participating in the distribution of shares to the broader market. Thus it may be more fruitful for researchers to explore whether syndicate members and/or prominent institutional investors are systematically excluded from future deals in the aftermath of perceived violations of *informal* penalty bids of the sort envisioned by our model.

## 5 Model Robustness

Our key assumptions thus far surround how information regarding the IPO continues to arrive sequentially in the secondary market. In our setting the issuer benefits from staging the IPO because the expected arrival of date 2 informed investors forces date 1 investors to bid more aggressively. Thus the surplus extracted by secondary market investors is diminished. The dependence of our results on this assumption suggests the issuer might simply delay the offering until date 2 and then conduct a uniform price auction. For example, Bulow and Klemperer (1996) show that in a private value, second-price auction setting it is optimal to delay the sale to include more participants if there is no cost to doing so.

We examine this potential challenge to the robustness of our argument for a staged, declining price mechanism by altering the model structure in two respects. First, we assume that investors have private values in the sense that investor  $i$ 's valuation,  $v_i = s_i$ , is uniformly, identically and independently distributed on  $[0, 1]$ . Second, we assume that the market clearing mechanism is an ascending price (English) auction at each of date 1 and date 2. At date 1, a reservation price,  $p_1$ , may be set but at date 2 all remaining shares must be sold. Assuming there are  $g$  shares for sale, the auctioneer continuously raises the price from its lowest possible value until only  $g$  bidders remain. Each of the  $g$  bidders pays the price at which  $g + 1$  bidders remained in the auction. In this setting we examine the consequences of costly delay of sales to date 2.

## 5.1 Equilibrium Analysis

At date 2, it is a standard result that investors participating in the English auction bid their true valuations. Dropping out before the price rises to this level ensures a zero payoff and remaining in the auction beyond one's true valuation ensures a negative payoff.

Before proceeding to date 1 it is necessary to introduce additional notation. Let  $B_1$  denote the set of date 1 buyers and  $B_2$  the set of date 2 buyers.  $B_1$  contains  $d_1$  investors and  $B_2$  contains  $n - d_1$  investors.  $I$  represents total set of investors.

At date 1, investors face a trade-off. Remaining in the auction ensures the investor of receiving a share at price  $p_1$  because there are more shares for sale than there are date 1 investors ( $m_1 < n$ ). Waiting until date 2 to bid for shares places the investor in competition with new arrivals for any remaining shares. Thus investor  $i$  participates in the date 1 auction if and only if

$$v_i - p_1 \geq E\{\Pr(s^{(n-d_1^{-i})}(B_2 \setminus i) \leq v_i | d_1^{-i}) E[v_i - s^{(n-d_1^{-i})}(B_2 \setminus i) | s^{(n-d_1^{-i})}(B_2 \setminus i) \leq v_i | d_1^{-i}]\} \quad (6)$$

where  $s^{(n-d_1^{-i})}(B_2 \setminus i)$  is the  $n - d_1^{-i}$  order statistic or the  $n - d_1^{-i}$  highest valuation among investors except  $i$  remaining in  $B_2$  (where  $B_2 \setminus i$  means " $B_2$  exclude  $i$ "). For any given  $d_1^{-i}$ , if  $s^{(n-d_1^{-i})}(B_2 \setminus i) \leq v_i$ , investor  $i$  is allocated one share in the date 1 auction and pays  $s^{(n-d_1^{-i})}(B_2 \setminus i)$ ; otherwise he receives no allocation and thus zero payoff. Therefore the right hand side is investor  $i$ 's expected payoff if he bids at date 2. The left hand side is his payoff if he bids at date 1. By the usual revealed preference argument, it can be

shown that investor  $i$ 's equilibrium strategy is monotonic at date 1; if an investor  $i$  with valuation  $v_i$  bids at date 1, investors with valuation  $v'_i > v_i$  also bid. Thus investor  $i$ 's strategy is characterized by a  $v_{1i}$ , such that if  $v_i > v_{1i}$ , he bids at date 1. Focusing on symmetric equilibria where  $v_{1i} = v_1$ , an investor  $i$  with date 1 valuation  $v_1$  is indifferent between bidding at date 1 or date 2 implying that his incentive-compatibility condition binds:

$$v_1 - p_1 = E\{\Pr(s^{(n-d_1^{-i})}(B_2 \setminus i) \leq v_1 | d_1^{-i}) E[v_1 - s^{(n-d_1^{-i})}(B_2 \setminus i) | s^{(n-d_1^{-i})}(B_2 \setminus i) \leq v_1 | d_1^{-i}]\}. \quad (7)$$

Note that date 1 demand equal to  $d_1^{-i}$  means that  $d_1^{-i}$  buyers have valuations higher than  $v_1$ , the remaining  $m_1 - d_1^{-i}$  investors have valuations lower than  $v_1$ .

For the sake of simplicity, we assume that any delay costs can be represented by a cost  $c$  per share remaining unsold at the beginning of both date 1 and date 2. This means that if the sale of all  $n$  shares is put off entirely until date 2, the issuer incurs a cost equal to  $2nc$ . The number of shares sold at date 1,  $d_1$ , reduces the total cost by  $d_1c$ . Given the investors' strategy, the issuer chooses  $p_1$  to maximize his payoff  $P$  (expected revenue net of selling costs) from the IPO:

$$P = E[p_1 d_1 + (n - d_1) E[s^{(n-d_1+1)}(B_2) | d_1] - nc - (n - d_1)c]. \quad (8)$$

The first term reflects revenue from selling shares at date 1. The second term reflects revenue at date 2. The date 2 price,  $p_2$ , is  $s^{(n-d_1+1)}(B_2)$  which denotes the  $n - d_1 + 1$  order statistics for the valuations of investors in set  $B_2$ .

Because (7) specifies that  $p_1$  is a function of  $v_1$ , the issuer's strategy is equivalent to choosing  $v_1$  to maximize his payoff. Lemma 6 states that the issuer's payoff can be restated as a function of the marginal investor's date 1 valuation,  $v_1$ .

**Lemma 6**  $p_1(v_1) = v_1 - E[v_1 - s^{(n)}(I \setminus i) | s^{(n)}(I \setminus i) \leq v_1] \Pr(s^{(n)}(I \setminus i) \leq v_1)$ ,  $i \in I_1$ .

$s^{(n)}(I \setminus i)$  denotes the  $n$  th order statistics for the valuations of investors in set  $I \setminus i$ . With lemma 6, we can write down the issuer's payoff as a function of  $v_1$ . The following proposition characterizes the benchmark case where the issuer chooses the payoff maximizing  $v_1$  assuming zero delay costs ( $c = 0$ ).

**Proposition 7** *If  $c = 0$ , then  $v_1 = 1$  in equilibrium; if delay costs are zero, there is no staged selling and all shares are sold at date 2.*

Proposition 7 is consistent with Bulow and Klemperer (1996). By contrast, Proposition 8 demonstrates that positive delay costs provide the issuer with incentive to stage the sale of shares across the two periods.

**Proposition 8** *if  $c > 0$ , then  $v_1 < 1$  in equilibrium and with positive probability some shares are sold at date 1.*

Intuition for these propositions follows from a standard result in mechanism design [Maskin and Riley (1989)] implying here that the issuer's revenue is a function only of the share allocations and the payoff received by the lowest valuation investor. The proof of proposition 7 provided in the appendix demonstrates that the optimal allocation strategy calls for allocating  $n$  shares to investors with the  $n$  highest valuations. The allocation strategy is implemented by setting  $v_1 = 1$ . Any deviation from this strategy yields the issuer less revenue.

Intuition for proposition 8 is gained by comparing the revenue lost when deviating from the optimal allocation strategy to the cost savings in staging the IPO rather than delaying the entire sale until date 2. Consider first the revenue loss from reducing  $v_1$  from 1 by a small amount,  $\epsilon$ . Deviations from the revenue maximizing allocation occur only when (i) there is a date 1 investor with valuation greater than  $v_1$  and (ii) his valuation is lower than the  $n$ th highest valuation of all other investors. The probability of either event is at most the same order of magnitude as  $\epsilon$ . Their joint probability, and thus the revenue loss, is of a higher order of magnitude than  $\epsilon$ . But if there are  $d_1$  date 1 buyers, induced by setting  $v_1 < 1$ , delay costs are reduced by  $d_1 c$ . Since the probability of  $d_1 > 0$  is of the same order of magnitude as  $\epsilon$ , so is the cost reduction. Therefore, with positive delay costs, cost reductions outweigh revenue losses for (an optimal)  $v_1$  less than 1.

The final proposition establishes the range of delay costs over which a staged offering is optimal and thus a declining price schedule arises in a private value, ascending price auction setting.

**Proposition 9** *There exist a  $\bar{c}$ , such that if  $0 < c < \bar{c}$ ,  $p_1 > E[p_2]$ .*

When  $v_1 = 1$ ,  $p_1(1) > E[p_2(v_1 = 1)]$  because the date 1 price must be high enough to discourage all investors from bidding at date 1. When the delay cost  $c$  is small, the analysis implies that  $v_1$  is close to

1. In this case,  $p_1$  is close to  $p_1(1)$  and  $E[p_2]$  is close to  $E[p_2(v_1 = 1)]$ . Since  $p_1(1) > E[p_2(v_1 = 1)]$ , we can conclude that if  $c$  is small enough, the equilibrium price is expected to decrease. Having established a decreasing price schedule, all of the analysis in the dynamic model of the preceding section applies to the private value, ascending price auction setting.

Finally, note that these results do not depend on the uniform distribution assumption. It is straightforward to show that they hold true for more general distributional assumptions under the usual mild assumption that  $v - \frac{1-F(v)}{f(v)}$  increases in  $v$ .

## 6 Conclusion

In this paper we develop a model in which investment banks and institutional investors collaborate in smoothing an IPO's transition to secondary market trading. Their intervention promotes welfare under the assumptions that significant new information arrives in the market in the immediate aftermath of the IPO and that reducing information rent diminishes the issuer's financing costs. Under these assumptions, it is optimal to stage the offering and suboptimal to commit to selling shares at a uniform price. In essence, the optimal strategy yields an economic rationale for secondary market price stabilization for IPOs carried out via a well-coordinated network of repeat institutional investors.

Regulatory constraints requiring uniform pricing of IPO shares prevent direct implementation of the optimal strategy. Temporary placement of shares at a uniform price with institutional investors who then sell shares in the secondary market according to the optimal strategy enables indirect implementation of the optimal strategy. Indirect implementation is complicated by the fact that any single institutional investor has incentive to break ranks and flip its shares as quickly as possible.

In a dynamic setting the threat of exclusion from future IPOs can be sufficient to sustain institutional cooperation in the optimal distribution of IPO shares to secondary market investors. Issuers contract with investment banks in this setting because the latter's repeated dealing with institutional investors lends credibility to the threat of exclusion. This implicit threat resembles a less easily observed but Draconian version of the formal penalty bids more commonly, but infrequently, imposed on retail investors.

The optimal strategy yields price dynamics consistent with the unusual patterns associated with IPOs. Specifically, the model predicts "underpricing" at the offering, price increases in the immediate secondary

market, and declining prices as the secondary market settles into "normal" trading patterns. The model also provides a rationale for concentration of initial allocations among institutional investors that complements that arising from the bookbuilding literature.

Our results are robust to a setting in which the market clearing mechanism is an ascending price auction. This finding speaks both to the general nature of our results and to the debate surrounding whether and how electronic auctions might displace traditional practices for selling IPOs. Our model suggests that even if auctions were to displace bookbuilding practices, issuers would have incentive to stage the auction and limit participation if they expect their IPO to trigger subsequent production of information. If such information production is expected to bear heavily on immediate secondary market trading, the optimal strategy will likely reflect a demand for secondary market price stabilization of the sort envisioned by the SEC as beneficial to the distribution of IPOs.

## 7 Appendix: Proofs of Propositions

**Proof of Proposition 2:** First, we demonstrate that investors will not deviate from their equilibrium strategies (parts (i) and (ii) in Proposition 2) given the issuer's pricing rule.

If a date 1 type  $h$  investor,  $i$ , bids at date 2, his payoff is

$$E\left\{\frac{n-d_1^{-i}}{m_1+m_2-d_1^{-i}}[E[V|s_i=h, d_1^{-i}] - E[V|s_j=l, d_1^{-i}]]\right\} = \alpha(h-l)\frac{2-\frac{1}{2}b}{2-b}E\left[\frac{n-d_1^{-i}}{m_1+m_2-d_1^{-i}}\right]. \quad (9)$$

This follows from

$$E[V|s_i=h, d_1^{-i}] = \alpha[h + d_1^{-i}h + (m_1 - d_1^{-i} - 1)E[s_g|d_1^g=0] + \frac{1}{2}(h+l)m_2], \quad (10)$$

and

$$E[V|s_j=l, d_1^{-i}] = \alpha[d_1^{-i}h + (m_1 - d_1^{-i})E[s_g|d_1^g=0] + \frac{1}{2}(h+l)(m_2-1) + l], \quad (11)$$

where  $d_1^g$  denotes a date 1 bid by investor  $g$  ( $d_1^g = 1$  if he bids and  $d_1^g = 0$  if he does not) and  $\Pr[s_g = h|d_1^g = 0] = \frac{\frac{1}{2}(1-b)}{1-\frac{1}{2}b}$ .

If, instead, the date 1 type  $h$  investor bids at date 1, his payoff is  $E[V|s_i=h] - p_1^* = \alpha(h-l)\frac{2-\frac{1}{2}b}{2-b}E\left[\frac{n-d_1^{-i}}{m_1+m_2-d_1^{-i}}\right]$  which is identical to the payoff for bidding at date 2 given in (9). Thus the date 1 type  $h$  investor is indifferent between bidding at date 1 and date 2.

The payoff faced by a date 1  $l$  type investor,  $r$ , who deviates from the equilibrium strategy by bidding at date 1 is  $E[V|s_r=l] - p_1^* = \alpha(h-l)\frac{2-\frac{1}{2}b}{2-b}E\left[\frac{n-d_1^{-i}}{m_1+m_2-d_1^{-i}}\right] - \frac{h-l}{m_1+m_2} < \alpha(h-l)\left(\frac{\frac{1}{2}b}{2-b}\right)E\left[\frac{n-d_1^{-r}}{m_1+m_2-d_1^{-r}}\right]$ . His payoff from not bidding at date 1 is  $E\left\{\frac{n-d_1}{m_1+m_2-d_1}[E[V|s_r=l, d_1] - E[V|s_j=l, d_1]]\right\} = \alpha(h-l)\frac{\frac{1}{2}b}{2-b}E\left[\frac{n-d_1^{-r}}{m_1+m_2-d_1^{-r}}\right]$  which follows from  $d_1 = d_1^{-r}$  conditional on  $s_r = l$  in equilibrium. Thus date 1 type  $l$  investors will not deviate from their equilibrium strategy of not bidding at date 1.

Finally, it is obvious that date 2 type  $h$  investors will bid at date 2 while date 2 type  $l$  investors are indifferent between bidding and not bidding at  $p_2$ .

Next, we show that the issuer's equilibrium pricing rule is optimal given investors' strategies. Since the issuer must complete the sale of shares by date 2, the optimal date 2 price must induce bids from all date 2 investors. If it does not, there is a positive probability that the distribution of shares will not be completed

at date 2. Thus the optimal  $p_2 = E[V|s_j = l, d_1], \forall j \in I_2$ . This establishes the date 2 equilibrium price given part (iii) of Proposition 2.

Establishing the date 1 equilibrium price requires consideration of four cases:

*Case I:  $0 \leq b \leq 1$ .*

In this case, we establish the optimal bidding strategy induced by the issuer given in part (i) of Proposition 2. All date 1 type  $h$  investors follow a mixed strategy by bidding with probability  $b$ . Date 1 type  $l$  investors do not bid. Given  $p_2$ , the date 1 price,  $p_1$ , must satisfy:

$$\begin{aligned}
E[V|s_i = h] - p_1 &= E\left\{\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}[E[V|s_i = h, d_1^{-i}] - E[V|s_j = l, d_1 = d_1^{-i}]]\right\} \\
&= \left[\frac{1}{2}(h - l)\alpha + (h - l)\alpha\left(1 - \frac{\frac{1}{2}(1 - b)}{1 - \frac{1}{2}b}\right)\right]E\left[\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}\right] \\
&= \alpha(h - l)\frac{2 - \frac{1}{2}b}{2 - b}E\left[\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}\right]. \tag{12}
\end{aligned}$$

Since  $E\left[\frac{n - d_1^{-i}(b)}{m_1 + m_2 - d_1^{-i}(b)}\right]$  is continuous in  $b$ ,  $\alpha(h - l)\frac{2 - \frac{1}{2}b}{2 - b}E\left[\frac{n - d_1^{-i}(b)}{m_1 + m_2 - d_1^{-i}(b)}\right]$  is continuous in  $b$ . Therefore, for any  $p_1$  between  $p_1^*$  and  $E[V|s_i = h] - \frac{n}{m_1 + m_2}\alpha(h - l)$ , there exists a bidding probability  $b(p_1)$  such that (12) is satisfied.

The issuer's objective is to set  $b(p_1)$  to minimize investor surplus: since this is a constant sum game (the sum of the *ex ante* expected payoffs of all agents is  $E[V]n$ ), the issuer's payoff is decreasing in investor surplus. Investor surplus is the total surplus captured by four investor types. Date 1 type  $h$  investors realize surplus equal to  $r_1^h = \alpha(h - l)\frac{2 - \frac{1}{2}b}{2 - b}E\left[\frac{n - d_1^{-i}(b)}{m_1 + m_2 - d_1^{-i}(b)}\right]$ .

Surplus for date 1 type  $l$  investors is

$$\begin{aligned}
r_1^l &= E\left\{[E[V|s_i = l, d_1] - p_2(d_1)]\frac{n - d_1(b)}{m_1 + m_2 - d_1(b)}\right\} \\
&= \left[\frac{1}{2}(h - l)\alpha - \alpha(h - l)\frac{\frac{1}{2}(1 - b)}{1 - \frac{1}{2}b}\right]E\left[\frac{n - d_1(b)}{m_1 + m_2 - d_1(b)}\right] \\
&= \alpha(h - l)\frac{\frac{1}{2}b}{2 - b}E\left[\frac{n - d_1(b)}{m_1 + m_2 - d_1(b)}\right];
\end{aligned}$$

Surplus for date 2 type  $h$  investors is  $r_2^h = \alpha(h - l)E\left[\frac{n - d_1(b)}{m_1 + m_2 - d_1(b)}\right]$ . Date 2 type  $l$  investors gain no surplus ( $r_2^l = 0$ ).

Summing across the four investor types yields total *ex ante* investor surplus  $S = m_1(\frac{1}{2}r_1^h + \frac{1}{2}r_1^l) + m_2(\frac{1}{2}r_2^h + \frac{1}{2}r_2^l)$ .

Thus, the issuer optimally sets the date 1 price by setting  $b^*$  such that

$$b^* \in \arg \min_{b \in [0,1]} \left\{ \frac{m_1}{2} \alpha(h-l) \frac{2}{2-b} E \left[ \frac{n - d_1^{-i}(b)}{m_1 + m_2 - d_1^{-i}(b)} \right] + \frac{m_2}{2} \alpha(h-l) E \left[ \frac{n - d_1(b)}{m_1 + m_2 - d_1(b)} \right] \right\} \quad (13)$$

$$\text{Case II: } p_1 > \underset{b}{\text{Max}} E[V|s_i = h] - \alpha(h-l) \frac{2 - \frac{1}{2}b}{2-b} E \left[ \frac{n - d_1^{-i}(b)}{m_1 + m_2 - d_1^{-i}(b)} \right].$$

This case rules out a date 1 price exceeding the equilibrium price given in part (iii), Proposition 2. We first assume and then prove that date 1 investors will not bid in this case.

At date 2, the issuer sets the price at  $p_2 = E[V|s_i = l]$  such that all participating investors bid at date 2. Since at date 1  $p_1 > E[V|s_i = h] - \alpha(h-l) E \left[ \frac{n - d_1^{-i}(0)}{m_1 + m_2 - d_1^{-i}(0)} \right] = E[V|s_i = h] - \frac{n}{m_1 + m_2} \alpha(h-l)$ ,  $E[V|s_i = h] - p_1 < \frac{n}{m_1 + m_2} \alpha(h-l)$  implying that date 1 investors will not bid at date 1. In this case, the issuer's revenue, derived from selling the entire offering at date 2, is  $E[V|s_i = l]n$ . This payoff is (weakly) less than the *case I* payoff where  $b = 0$  and thus (weakly) less than the issuer's equilibrium payoff.

$$\text{Case III: } E[V|s_i = l] < p_1 < \underset{b}{\text{Min}} E[V|s_i = h] - \alpha(h-l) \frac{2 - \frac{1}{2}b}{2-b} E \left[ \frac{n - d_1^{-i}(b)}{m_1 + m_2 - d_1^{-i}(b)} \right].$$

In this case, all date 1 type  $h$  investors bids at date 1 with probability 1, and all type  $l$  investors do not and thus similar to case I where  $b = 1$ . The issuer's payoff is weakly less than the equilibrium payoff since it is weakly less than in the case where  $b = 1$ .

$$\text{Case IV: } p_1 \leq E[V|s_i = l].$$

Since  $p_2 = E[V|s_j = l, d_1]$ ,  $\forall j \in I_2$ , the issuer's payoff is  $p_1 E[d_1] + E[p_2(d_1)(n - d_1)]$ . The usual revealed preference argument shows that a date 1 type  $h$  investor bids at date 1 with higher probability than if he has signal  $l$ . Therefore,  $p_2(d_1)$  is increasing in  $d_1$  and  $\text{cov}[p_2(d_1), (n - d_1)] \leq 0$ . This implies  $p_1 E[d_1] + E[p_2(d_1)(n - d_1)] \leq p_1 E[d_1] + E[p_2(d_1)] E[(n - d_1)] \leq E[V|s_j = l]n$ . The last inequality follows from  $E[p_2(d_1)] = E[V|s_j = l]$  and  $E[V|s_i = l] \geq p_1$ . Thus the issuer's payoff in this case is (weakly) less than that in case I where  $b = 0$  and thus (weakly) less than his equilibrium payoff.

The conditions for cases I, III, and IV are not mutually exclusive. We settle the ambiguity off the equilibrium path by specifying that if  $p_1 \leq E[V|s_i = l]$ , case IV applies. For,  $p_1 > E[V|s_i = l]$ , case I applies whenever possible and otherwise case III applies.

The preceding analysis proves (i), (ii), and (iii). Since  $E[p_2^*] = E[V|s_j = l]$ , part (iv) is true by the proof in case IV. Part (v) is true if

$$\alpha(h-l) \frac{2 - \frac{1}{2}b^*}{2 - b^*} E\left[\frac{n - d_1^{-i}(b^*)}{m_1 + m_2 - d_1^{-i}(b^*)}\right] - \frac{1}{2}\alpha(h-l) \leq 0,$$

which is true iff

$$\frac{2 - \frac{1}{2}b^*}{2 - b^*} E\left[\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}\right] \leq \frac{1}{2}.$$

Since  $\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}$  is decreasing in  $d_1^{-i}$  and  $\frac{2 - \frac{1}{2}b^*}{2 - b^*}$  is increasing in  $b^*$ , a sufficient condition for the above inequality is that  $3\frac{n}{m_1 + m_2} \leq 1$ , which is  $m_2 \geq 3n - m_1$ .

**Proof of Proposition 4:** We begin by proving the following lemma:

*Lemma 1:* The bidding strategy for date 1 type h investors is symmetric:  $b_i(s_i = h) = b_j(s_j = h)$ .

Suppose this is not true. Let  $b_1^i$  be the largest probability with which date 1 type  $h$  investors bid and  $b_1^j$  be the smallest. In equilibrium,  $b_1^i > b_1^j$ . We simplify the notation by letting  $b_i = b_1^i$  and  $b_j = b_1^j$  and note that  $b_i$  can be one and  $b_j$  can be zero. The incentive compatibility conditions for  $i$  and  $j$  are

$$E[V|s_i = h] - p_1 \geq E\left\{\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}[E[V|s_i = h, d_1^{-i}] - p_2(d_1 = d_1^{-i})]\right\}, \quad q \in I_2, \quad (14)$$

$$E[V|s_j = h] - p_1 \leq E\left\{\frac{n - d_1^{-j}}{m_1 + m_2 - d_1^{-j}}[E[V|s_j = h, d_1^{-j}] - p_2(d_1 = d_1^{-j})]\right\}, \quad q \in I_2. \quad (15)$$

Denote  $d_1^{-ij}$  to be the number of bids at date 1 from all date 1 investors except  $i$  and  $j$ . Conditional on  $d_1^{-ij}$ , the right hand side of (14) is

$$\begin{aligned} E\left\{\frac{n - d_1^{-i}}{m_1 + m_2 - d_1^{-i}}[E[V|s_i = h, d_1^{-i}] - E[V|s_q = l, d_1 = d_1^{-i}]|d_1^{-ij}]\right\} &= \Pr(d_1^j = 1) \left\{\frac{n - d_1^{-ij} - 1}{m_1 + m_2 - d_1^{-ij} - 1}\right. \\ &\quad \times [E[V|s_i = h, d_1^{-ij}, d_1^j = 1] - p_2(d_1 = d_1^{-ij} + 1)] \left. + \Pr(d_1^j = 0) \left\{\frac{n - d_1^{-ij}}{m_1 + m_2 - d_1^{-ij}}\right.}\right. \\ &\quad \left. \times [E[V|s_i = h, d_1^{-ij}, d_1^j = 0] - p_2(d_1 = d_1^{-ij})]\right\}. \end{aligned}$$

Similarly, the right hand side of (15) is

$$\begin{aligned}
E\left\{\frac{n-d_1^{-j}}{m_1+m_2-d_1^{-j}}[E[V|s_j = h, d_1^{-j}] - E[V|s_q = l, d_1 = d_1^{-j}]]|d_1^{-ij}\right\} &= \Pr(d_1^i = 1)\left\{\frac{n-d_1^{-ij}-1}{m_1+m_2-d_1^{-ij}-1}\right. \\
&\times [E[V|s_i = h, d_1^{-ij}, d_1^i = 1] - p_2(d_1 = d_1^{-ij} + 1)]\left\} + \Pr(d_1^j = 0)\left\{\frac{n-d_1^{-ij}}{m_1+m_2-d_1^{-ij}}\right. \\
&\times [E[V|s_i = h, d_1^{-ij}, d_1^i = 0] - p_2(d_1 = d_1^{-ij})]\left\}.
\end{aligned}$$

The several expectations are:

$$\begin{aligned}
E[V|s_i = h, d_1^{-ij}, d_1^j = 1] &= \alpha\{h + E[s_j|d_1^j = 1]\} + \sum_{g \in I_1 \setminus i, j} E[s_g|d_1^{-ij}] \\
&= \alpha\{h + h + \sum_{g \in I_1 \setminus i, j} E[s_g|d_1^{-ij}]\}.
\end{aligned} \tag{16}$$

$E[s_j|d_1^j = 1] = h$  since only  $s_j = h$  would bid;

$$\begin{aligned}
E[V|s_i = h, d_1^{-ij}, d_1^j = 0] &= \alpha\{h + E[s_j|d_1^j = 0]\} + \sum_{g \in I_1 \setminus i, j} E[s_g|d_1^{-ij}] \\
&= \alpha\left\{h + \frac{\frac{1}{2}(1-b_j)}{1-\frac{1}{2}b_j}(h-l) + l + \sum_{g \in I_1 \setminus i, j} E[s_g|d_1^{-ij}]\right\},
\end{aligned} \tag{17}$$

since  $\Pr(s_j = h|d_1^j = 0) = \frac{\frac{1}{2}(1-b_j)}{1-\frac{1}{2}b_j}$ ;

$$E[V|s_j = h, d_1^{-ij}, d_1^i = 1] = \alpha\{h + h + \sum_{g \in I_1 \setminus i, j} E[s_g|d_1^{-ij}]\}; \tag{18}$$

and

$$E[V|s_j = h, d_1^{-ij}, d_1^i = 0] = \alpha\left\{h + \frac{\frac{1}{2}(1-b_i)}{1-\frac{1}{2}b_i}(h-l) + l + \sum_{g \in I_1 \setminus i, j} E[s_g|d_1^{-ij}]\right\}. \tag{19}$$

Comparing (17) with (19), and using the fact that,  $\frac{\frac{1}{2}(1-b_i)}{1-\frac{1}{2}b_i} < \frac{\frac{1}{2}(1-b_j)}{1-\frac{1}{2}b_j}$ , yields (i)  $E[V|s_j = h, d_1^{-ij}, d_1^i = 0] > E[V|s_i = h, d_1^{-ij}, d_1^j = 0]$ . Comparing (16) with (18) yields (ii)  $E[V|s_i = h, d_1^{-ij}, d_1^j = 1] = E[V|s_j = h, d_1^{-ij}, d_1^i = 1]$ . Finally, (iii)  $\Pr(d_1^j = 1) = \frac{1}{2}b_j < \frac{1}{2}b_i = \Pr(d_1^i = 1)$ .

Next we show that (iv)  $E[V|s_i = h, d_1^{-ij}, d_1^j = 1] - p_2(d_1 = d_1^{-ij} + 1) \leq E[V|s_i = h, d_1^{-ij}, d_1^j = 0] - p_2(d_1 = d_1^{-ij})$ . This is true if and only if  $E[V|s_i = h, d_1^{-ij}, d_1^j = 1] - E[V|s_i = h, d_1^{-ij}, d_1^j = 0] \leq$

$p_2(d_1 = d_1^{-ij} + 1) - p_2(d_1 = d_1^{-ij})$ . The left hand side of the inequality is  $\alpha(h-l)\frac{\frac{1}{2}}{1-\frac{1}{2}b_j}$  from (17) and (16).

The right hand side equals

$$\begin{aligned} E[V|d_1 = d_1^{-ij} + 1] - E[V|d_1 = d_1^{-ij}] &= \sum_{q \in I_1} \Pr(d_1^{-q} = d_1^{-ij}) \{E[V|d_1^q = 1, d_1^{-q} = d_1^{-ij}] - E[V|d_1^q = 0, d_1^{-q} = d_1^{-ij}]\} \\ &= \sum_{q \in I_1} \Pr(d_1^{-q} = d_1^{-ij}) [\alpha(h-l)\frac{\frac{1}{2}}{1-\frac{1}{2}b_q}]. \end{aligned}$$

Since  $b_j \leq b_q$ ,  $\alpha(h-l)\frac{\frac{1}{2}}{1-\frac{1}{2}b_q} \geq \alpha(h-l)\frac{\frac{1}{2}}{1-\frac{1}{2}b_j}$ , (iv) follows.

Assembling (i)-(iv) and using the fact that  $\frac{n-d_1^{-ij}-1}{m_1+m_2-d_1^{-ij}-1} < \frac{n-d_1^{-ij}}{m_1+m_2-d_1^{-ij}}$ , yields

$$\begin{aligned} &E\left\{\frac{n-d_1^{-i}}{m_1+m_2-d_1^{-i}}[E[V|s_i = h, d_1^{-i}] - E[V|s_q = l, d_1 = d_1^{-i}]]|d_1^{-ij}\right\} \\ &\stackrel{\text{(iii) and (iv)}}{\geq} \Pr(d_1^i = 1) \left\{\frac{n-d_1^{-ij}-1}{m_1+m_2-d_1^{-ij}-1}[E[V|s_i = h, d_1^{-ij}, d_1^j = 1] - p_2(d_1 = d_1^{-ij} + 1)]\right\} \\ &+ \Pr(d_1^i = 0) \left\{\frac{n-d_1^{-ij}}{m_1+m_2-d_1^{-ij}}[E[V|s_i = h, d_1^{-ij}, d_1^j = 0] - p_2(d_1 = d_1^{-ij})]\right\} \\ &\stackrel{\text{(i) and (ii)}}{>} \Pr(d_1^i = 1) \left\{\frac{n-d_1^{-ij}-1}{m_1+m_2-d_1^{-ij}-1}[E[V|s_j = h, d_1^{-ij}, d_1^i = 1] - p_2(d_1 = d_1^{-ij} + 1)]\right\} \\ &+ \Pr(d_1^i = 0) \left\{\frac{n-d_1^{-ij}}{m_1+m_2-d_1^{-ij}}[E[V|s_j = h, d_1^{-ij}, d_1^i = 0] - p_2(d_1 = d_1^{-ij})]\right\} \\ &= E\left\{\frac{n-d_1^{-j}}{m_1+m_2-d_1^{-j}}[E[V|s_j = h, d_1^{-j}] - E[V|s_q = l, d_1 = d_1^{-j}]]|d_1^{-ij}\right\}. \end{aligned}$$

Together with (14) and (15), this inequality implies

$$E[V|s_i = h] - p_1 > E[V|s_j = h] - p_1,$$

which is a contradiction since  $E[V|s_i = h] = E[V|s_j = h]$ . Therefore we have proved lemma 1.

Since the seller has to induce at least  $n - m_1$  date 2 investor to bid at date 2 for sure, it follows that some type  $l$  investors must to bid in equilibrium or that  $p_2 \leq E[V|s_j = l, d_1]$ ,  $\exists j \in I_2$ . Integrating out  $d_1$ , yields  $E[p_2] \leq E[V|s_j = l]$ . There are three cases at date 1:

(i) nobody bids at date 1 and everybody bids a date 2. The equilibrium payoff to the seller is  $E[p_2]n$ , or less than his payoff in the equilibrium described in Proposition 2.

(ii) Some type  $l$  investors bid at date 1 in which case  $p_1 \leq E[V|s_i = l]$ . The same argument as that used in case IV in the proof of Proposition 2 shows that the seller's payoff in any equilibrium with  $p_1 \leq E[V|s_i = l]$  is strictly lower than the equilibrium payoff described in Proposition 2.

(iii) Only type  $h$  investors bid at date 1. By lemma 1, we know that this case is exactly case I studied in the proof of Proposition 2. Therefore, we can conclude that the optimal equilibria is as described in Proposition 2

**Proof of proposition 5:** The investment bank treats all institutional investors symmetrically in equilibrium such that it selects randomly which institutional investors will sell their shares to meet date 1 demand  $d_1$ . Thus in a stage game, each institutional investor has expected payoff  $\frac{1}{k}(E[V]n - S - np_0)$  where  $E[V]n - S$  is the total expected revenue institutional investors receive in equilibrium.

Now suppose that in period  $\tau$ , an institutional investor sells shares at date 1 contrary to the investment bank's plan. The institutional investor obtains an extra payoff of  $(p_1^* - \frac{E[V]n - S}{n})\frac{n}{k}$  but forfeits all future expected payoffs.

Since

$$\begin{aligned} E[V]n - S &= E[d_1 p_1^* + (n - d_1) p_2^*(d_1)] \\ &= E[d_1 p_1^*] + E[(n - d_1)] E[p_2^*(d_1)] + Cov[n - d_1, p_2^*(d_1)] \\ &< E[d_1 p_1^*] + E[(n - d_1)] E[p_2^*(d_1)] \\ &< p_1^* n, \end{aligned}$$

the first inequality follows because  $n - d_1$  and  $p_2^*(d_1)$  are negatively correlated. So we have  $p_1^* - \frac{E[V]n - S}{n} > 0$ . Therefore, the institutional investor will not deviate if

$$\sum_{\tau=1}^{\infty} \delta^\tau \frac{1}{k} (E[V]n - S - np_0) \geq (p_1^* - \frac{E[V]n - S}{n}) \frac{n}{k} \quad (20)$$

or, alternatively, if

$$(E[V]n - S - np_0) \geq \frac{1 - \delta}{\delta} (p_1^* - \frac{E[V]n - S}{n}) n$$

The investment banks sets the optimal offer price  $p_0$  such that this inequality is saturated. Therefore,

$$p_0^* = (E[V] - \frac{S}{n}) - \frac{1 - \delta}{\delta} (p_1^* - \frac{E[V]n - S}{n}).$$

We assume  $\delta$  is high enough so that  $p_0^*$  is positive, i.e.,  $(E[V] - \frac{S}{n})\delta > (1 - \delta)(p_1^* - \frac{E[V]n - S}{n})$ . That is,

$$1 > \delta > \frac{p_1 - \frac{E[V]n - S}{n}}{E[V] - \frac{S}{n} + p_1 - \frac{E[V]n - S}{n}}.$$

If an institutional investor deviates, the investment bank has no incentive to continue including him in the IPO process since the investor's strategy is to flip his allocation in every offering. On the other hand, if the investment bank excludes the deviating investor forever, the investor's best response is to flip every time. Therefore, excluding the institutional investor forever is part of the equilibrium. Parts (i) to (v) follow directly from this analysis.

Finally, we show that the equilibrium maximizes the investment bank's payoff among the equilibria that implement the optimal staged equilibrium [part (vi)]. At period 0, in any equilibrium which implements the optimal stage equilibrium, summing incentive-compatibility constraints across all institutional investors yields

$$\sum_{\tau=1}^{\infty} \delta^{\tau} (E[V]n - S - np_0^{\tau}) \geq (p_1^* - \frac{E[V]n - S}{n})n + \sum_{q=1}^k v_q,$$

where  $v_q$  is the discounted payoff from future interaction with the investment bank if institutional investor  $q$  flips in period 0.

$v_q \geq 0$  since the institutional investor can always choose not to interact with the investment bank in the future. Therefore

$$\begin{aligned} n \sum_{\tau=1}^{\infty} \delta^{\tau} p_0^{\tau} &\leq \frac{\delta}{1 - \delta} (E[V]n - S) - (p_1^* - \frac{E[V]n - S}{n})n - \sum_{q=1}^k v_q \\ &\leq \frac{\delta}{1 - \delta} (E[V]n - S) - (p_1^* - \frac{E[V]n - S}{n})n. \end{aligned}$$

The investment bank's payoff at date 1 is  $n \sum_{\tau=1}^{\infty} \delta^{\tau-1} p_0^{\tau}$ , which is bounded by  $\frac{1}{1-\delta}(E[V]n - S) - \frac{1}{\delta}(p_1^* - \frac{E[V]n - S}{n})n$ . By the perfectness requirements of the equilibrium, we know that the investment bank's set of equilibrium payoffs at period 1 is the same as at period 0. Therefore, the investment bank's date 0

payoff is also bounded by  $\frac{1}{1-\delta}(E[V]n - S) - \frac{1}{\delta}(p_1^* - \frac{E[V]n-S}{n})n$ . It is straightforward to demonstrate that the above equilibrium implements the upper bound of the investment bank's payoff.

**Proof of Lemma 6:** It is sufficient to show that  $E[v_1 - s^{(n)}(I \setminus i) | s^{(n)}(I \setminus i) \leq v_1] \Pr[s^{(n)}(I \setminus i) \leq v_1] = E\{\Pr[s^{(n-d_1^{-i})}(B_2 \setminus i) \leq v_1 | d_1^{-i}] E[v_1 - s^{(n-d_1^{-i})}(B_2 \setminus i) | s^{(n-d_1^{-i})}(B_2 \setminus i) \leq v_1 | d_1^{-i}]\}$ . Because all bidders at date 1 have a higher valuation than  $v_1$ , for any  $s^{(n-d_1^{-i})}(B_2 \setminus i) \leq v_1$  and  $d_1^{-i}$ ,  $s^{(n)}(I \setminus i) = s^{(n-d_1^{-i})}(B_2 \setminus i)$ . That is,

$$v_1 - s^{(n)}(I \setminus i) = v_1 - s^{(n-d_1^{-i})}(B_2 \setminus i).$$

Integrating both sides on  $[0, v_1]$ , we have

$$\begin{aligned} & \int_0^{v_1} [v_1 - s^{(n-d_1^{-i})}(B_2 \setminus i)] dF(s^{(n-d_1^{-i})}(B_2 \setminus i) | d_1^{-i}) \\ &= \int_0^{v_1} [v_1 - s^{(n)}(I \setminus i)] dF(s^{(n)}(I \setminus i) | d_1^{-i}). \end{aligned}$$

$F(s^{(n-d_1^{-i})}(B_2 \setminus i) | d_1^{-i})$  is the cumulative distribution function of  $s^{(n-d_1^{-i})}(B_2 \setminus i)$  conditional on  $d_1^{-i}$ .  $F(s^{(n)}(I \setminus i) | d_1^{-i})$  is similarly defined. Integrating  $d_1^{-i}$  out, we have

$$\begin{aligned} E\{\Pr[s^{(n-d_1^{-i})}(B_2 \setminus i) \leq v_1 | d_1^{-i}] E[v_1 - s^{(n-d_1^{-i})}(B_2 \setminus i) | s^{(n-d_1^{-i})}(B_2 \setminus i) \leq v_1 | d_1^{-i}]\} \\ &= \sum_{d_1^{-i}=0}^{m_1-1} \Pr(d_1^{-i}) \int_0^{v_1} [v_1 - s^{(n)}(I \setminus i)] d \frac{F(s^{(n)}(I \setminus i), d_1^{-i})}{\Pr(d_1^{-i})} \\ &= \sum_{d_1^{-i}=0}^{m_1-1} \int_0^{v_1} [v_1 - s^{(n)}(I \setminus i)] dF(s^{(n)}(I \setminus i), d_1^{-i}) \\ &= \int_0^{v_1} [v_1 - s^{(n)}(I \setminus i)] dF(s^{(n)}(I \setminus i)). \end{aligned}$$

The first equation follows because  $F(s^{(n)}(I \setminus i) | d_1^{-i}) = \frac{F(s^{(n)}(I \setminus i), d_1^{-i})}{\Pr(d_1^{-i})}$ .

**Proof of Proposition 7:** The proof closely follows Maskin and Riley (1989). Denote  $F_i(v)$  the cumulative distribution of investor  $i$ 's value. In our model,  $F_i(v) = v$ . Applying the revelation principle, the seller's problem is to design a multi-unit auction  $\{q_i(v_i, v_{-i}), T_i(v_i, v_{-i})\}$ , where  $v_{-i}$  denotes all investors'

values except  $i$ ,  $q_i$  denotes the probability  $i$  will get a share, and  $T_i$  is the amount investor  $i$  pays to the issuer. Investor  $i$ 's equilibrium payoff is thus

$$\pi_i(\widehat{v}_i, v_i) \equiv E_{v_{-i}}[v_i q_i(\widehat{v}_i, v_{-i})] - T_i(\widehat{v}_i),$$

where  $T_i(\widehat{v}_i) \equiv E_{v_{-i}}[T_i(\widehat{v}_i, v_{-i})]$ .  $E_{v_{-i}}[\cdot]$  denotes expectation of other investors' values. In a Bayesian equilibrium, each agent expects all other players to follow the equilibrium truth-telling strategy so that expectations reflect truth-telling among other players. Truth-telling is incentive compatible if and only if

$$\pi_i(v_i, v_i) = \max_{\widehat{v}_i} \pi_i(\widehat{v}_i, v_i).$$

It is straight forward to check that the incentive-compatibility condition is equivalent to the monotonicity of  $E_{v_{-i}}[q_i(v_i, v_{-i})]$  and the first-order condition. The first-order condition implies that  $\frac{d\pi_i}{dv_i}(v_i, v_i) = E_{v_{-i}}[q_i(v_i, v_{-i})]$ . Therefore,

$$\pi_i(v_i, v_i) = \pi_i(0, 0) + \int_0^{v_i} E_{v_{-i}}[q_i(x, v_{-i})] dx.$$

Together with the definition of  $\pi_i(\widehat{v}_i, v_i)$ , we have

$$T_i(v_i) = E_{v_{-i}}[v_i q_i(v_i, v_{-i})] - \pi_i(0, 0) - \int_0^{v_i} E_{v_{-i}}[q_i(x, v_{-i})] dx$$

so the issuer's expected revenue from  $i$  is

$$T_i^e = \int_0^1 \{E_{v_{-i}}[v_i q_i(v_i, v_{-i})] - \pi_i(0, 0)\} dF_i(v_i) - \int_0^1 \int_0^{v_i} E_{v_{-i}}[q_i(x, v_{-i})] dx dF_i(v_i)$$

The second term can be rewritten as follows:

$$\begin{aligned}
& \int_0^1 \int_0^{v_i} E_{v_{-i}}[q_i(x, v_{-i})] dx dF_i(v_i) \\
&= \int_0^1 \int_x^1 E_{v_{-i}}[q_i(x, v_{-i})] dF_i(v_i) dx \\
&= \int_0^1 (1 - F_i(x)) E_{v_{-i}}[q_i(x, v_{-i})] dx \\
&= \int_0^1 \left( \frac{1 - F_i(v_i)}{f_i(v_i)} \right) E_{v_{-i}}[q_i(v_i, v_{-i})] dF_i(v_i)
\end{aligned}$$

The first equality follows from changing the order of integration (Fubini's Theorem). The last equality follows from changing the dummy  $x$  to  $v_i$  and using the fact that  $dF_i(v_i) = f_i(v_i)dv_i$ . Therefore,

$$\begin{aligned}
T_i^e &= \int_0^1 \{E_{v_{-i}}[v_i q_i(v_i, v_{-i})] - \pi_i(0, 0)\} dF_i(v_i) - \int_0^1 E_{v_{-i}} \left[ \left( \frac{1 - F_i(v_i)}{f_i(v_i)} \right) q_i(v_i, v_{-i}) \right] dF_i(v_i) \\
&= \int_0^1 E_{v_{-i}} \left\{ v_i q_i(v_i, v_{-i}) - \pi_i(0, 0) - \left( \frac{1 - F_i(v_i)}{f_i(v_i)} \right) q_i(v_i, v_{-i}) \right\} dF_i(v_i) \\
&= E[v_i q_i(v_i, v_{-i}) - \pi_i(0, 0) - \left( \frac{1 - F_i(v_i)}{f_i(v_i)} \right) q_i(v_i, v_{-i})].
\end{aligned}$$

The issuer's problem is thus to maximize total revenue

$$\max_{q_i(\mathbf{v})} R \equiv E \left\{ \sum_i^{m_1+m_2} \left[ \left( v_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \right) q_i(\mathbf{v}) - \pi_i(0, 0) \right] \right\} \quad (21)$$

where  $\mathbf{v} = (v_i, v_{-i})$ , subject to

$$\pi_i(0, 0) \geq 0 \quad (22)$$

$$E_{v_{-i}}[q_i(v_i, v_{-i})] \text{ non-decreasing} \quad (23)$$

$$\sum_i^{m_1+m_2} q_i(\mathbf{v}) = n. \quad (24)$$

The solution is clearly  $\pi_i(0, 0) = 0$ .  $q_i(\mathbf{v}) = 1$  if  $J(v_i) \equiv v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}$  is among the  $n$  highest  $J$ . Under the assumption that  $J(v_i)$  is increasing in  $v_i$  (which is true for the uniform distribution), (23) is satisfied and  $q_i(\mathbf{v}) = 1$  for the  $n$  highest  $v_i$ . That is, the optimal selling mechanism is to sell the shares to the  $n$  highest value investors.

In our model, by setting  $v_1 = 1$  so that all selling occurs at date 2, the issuer implements the optimal selling mechanism.

**Proof of Proposition 8:** First we define revenue  $R(v_1) = E[p_1(v_1)d_1 + (n - d_1)E[s^{(n-d_1+1)}(B_2)|d_1]]$ , and cost  $C(v_1) = E[nc + (n - d_1)c]$ . We must show that (i)  $\frac{dR}{dv_1}|_{v_1=1} = 0$ , and (ii)  $\frac{dC}{dv_1}|_{v_1=1}$ . Because the issuer's payoff is  $R(v_1) - C(v_1)$ , (i) and (ii) imply that in the neighborhood of  $v_1 = 1$ , lowering  $v_1$  increases the issuer's payoff. Thus we prove the desired results.

(i):  $\frac{dC}{dv_1}|_{v_1=1} > 0$ .

$$\begin{aligned} \frac{dC}{dv_1} &= \frac{d}{dv_1} \left[ \sum_{d_1=0}^{m_1} C_{m_1}^{d_1} (1 - v_1)^{d_1} (v_1)^{m_1-d_1} (2nc - d_1c) \right] \\ &= \sum_{d_1=0}^{m_1} C_{m_1}^{d_1} \frac{d}{dv_1} [(1 - v_1)^{d_1} (v_1)^{m_1-d_1}] (2nc - d_1c) \end{aligned}$$

At  $v_1 = 1$ , the derivative is

$$\begin{aligned} &= \sum_{d_1=0}^1 C_{m_1}^{d_1} \frac{d}{dv_1} [(1 - v_1)^{d_1} (v_1)^{m_1-d_1}] (2nc - d_1c) \\ &= cm_1 \\ &> 0. \end{aligned}$$

The first equation follows because for  $d_1 > 1$ ,  $\frac{d}{dv_1} [(1 - v_1)^{d_1} (v_1)^{m_1-d_1}]|_{v_1=1} = 0$ . So, all  $d_1 > 1$  terms vanish.

(ii)  $\frac{dR}{dv_1}|_{v_1=1} = 0$ . That is  $\lim_{\epsilon \downarrow 0} \frac{R(1) - R(1-\epsilon)}{\epsilon} = 0$

By (21) and the fact that  $\pi_i(0, 0) = 0$  for both  $v_1 = 1$  and  $v_1 = 1 - \epsilon$ ,

$$\begin{aligned} R(1) - R(1 - \epsilon) &= E \left\{ \sum_i^{m_1+m_2} [J(v_i)[q_i^1(\mathbf{v}) - q_i^{1-\epsilon}(\mathbf{v})] \right\} \\ &\leq E \left\{ \sum_i^{m_1+m_2} [|J(v_i)| \mathbf{1}_{[q_i^1(\mathbf{v}) \neq q_i^\epsilon(\mathbf{v})]}] \right\} \\ &\leq J(1)(m_1 + m_2) \Pr[q_i^1(\mathbf{v}) \neq q_i^\epsilon(\mathbf{v}), \exists i \in I]. \end{aligned}$$

where  $q_i^1(\mathbf{v})$  and  $q_i^{1-\epsilon}(\mathbf{v})$  denote  $i$ 's allocation when  $v_1 = 1$  and  $v_1 = 1 - \epsilon$ , respectively.  $\mathbf{1}_{[q_i^1(\mathbf{v}) \neq q_i^\epsilon(\mathbf{v})]}$  is the indicator function of  $q_i^1(\mathbf{v}) \neq q_i^\epsilon(\mathbf{v})$ . It takes value 1 if  $q_i^1(\mathbf{v}) \neq q_i^\epsilon(\mathbf{v})$  and 0 otherwise. The last inequality follows because  $J(1) \geq \max |J(v_i)|$  for the uniform distribution.

Since if there is an  $i$  such that  $q_i^1(\mathbf{v}) \neq q_i^\epsilon(\mathbf{v})$ , then there must exist a  $j \in I_1$  such that  $v_j > 1 - \epsilon$  and  $v_j < s^{(n)}(I \setminus j)$ . Thus

$$\begin{aligned} \Pr[q_i^1(\mathbf{v}) \neq q_i^\epsilon(\mathbf{v}), \exists i \in I] &\leq \Pr[v_j > 1 - \epsilon \text{ and } v_j < s^{(n)}(I \setminus j), \exists j \in I_1] \\ &= \Pr[v_j > 1 - \epsilon, \exists j \in I_1] \Pr[s^{(n)}(I \setminus j) \geq 1 - \epsilon] \\ &\leq [1 - (1 - \epsilon)^{m_1}] \left[ \sum_{r=n}^{m_1+m_2-1} C_{m_1+m_2-1}^r \epsilon^r (1 - \epsilon)^{m_1+m_2-1-r} \right]. \end{aligned}$$

Therefore,

$$\begin{aligned} 0 &\leq \lim_{\epsilon \downarrow 0} \frac{R(1) - R(1 - \epsilon)}{\epsilon} \leq \lim_{\epsilon \downarrow 0} J(1)(m_1 + m_2) \left[ \frac{1 - (1 - \epsilon)^{m_1}}{\epsilon} \right] \left[ \sum_{r=n}^{m_1+m_2-1} C_{m_1+m_2-1}^r \epsilon^r (1 - \epsilon)^{m_1+m_2-1-r} \right] \\ &= J(1)(m_1 + m_2) m_1 \lim_{\epsilon \downarrow 0} \left[ \sum_{r=n}^{m_1+m_2-1} C_{m_1+m_2-1}^r \epsilon^r (1 - \epsilon)^{m_1+m_2-1-r} \right] = 0 \end{aligned}$$

That is,  $\frac{dR}{dv_1} \Big|_{v_1=1} = 0$ .

**Proof of proposition 9:** First, we show that  $R(v_1)$  is increasing so that  $R(v_1) > R(v'_1)$  for  $v_1 > v'_1$ . We compare allocations for  $v_1$  and  $v'_1$  for a fixed  $v$ . Under  $v_1$ ,  $d_1$  date 1 investors with value higher than  $v_1$  get one share, the remaining highest  $n - d_1$  investors get one share each. The rest get zero. That is, for a given  $\mathbf{v}$ ,

$$\begin{aligned} R(v_1) &= \sum_{i \in B_1(v_1)} J(v_i) + \sum_{j \in \{\text{the highest } n-d_1 \text{ value}\}} J(v_j) \\ &= \sum_{i \in B_1(v_1)} J(v_i) + \max_{q_j(\mathbf{v})} \sum_{j \in I \setminus B_1(v_1)} J(v_j) q_j(\mathbf{v}) \end{aligned}$$

where  $B_1(v_1)$  is the set of date 1 buyers, subject to

$$\sum_{j \in I \setminus B_1(v_1)} q_j(\mathbf{v}) = n - d_1.$$

Optimality follows because  $J(v_j)$  is increasing in  $v_j$ . For the same  $\mathbf{v}$ , under  $v'_1$ , the  $d_1$  date 1 investors with value higher than  $v_1$  get one share too. So

$$R(v'_1) = \sum_{i \in B_1(v_1)} J(v_i) + \sum_{j \in I \setminus B_1(v_1)} J(v_j) q_j^{v'_1}(\mathbf{v}).$$

Since  $\sum_{j \in I \setminus B_1(v_1)} q_j^{v'_1}(\mathbf{v}) = n - d_1$ , we conclude that

$$\max_{q_j(\mathbf{v})} \sum_{j \in I \setminus B_1(v_1)} J(v_j) q_j(\mathbf{v}) \geq \sum_{j \in I \setminus B_1(v_1)} J(v_j) q_j^{v'_1}(\mathbf{v})$$

for any  $\mathbf{v}$ . Furthermore, it is straight forward to check that the inequality is strict for some  $\mathbf{v}$  with non-trivial probability measure. Therefore,  $R(v_1) > R(v'_1)$ .

Second, we show the desired result. For any small  $\epsilon > 0$ , we can find a  $\bar{c}$  such that for  $0 < c < \bar{c}$

$$R(1) - 2nc - R(1 - \epsilon) > 0.$$

Here  $\bar{c} = \frac{R(1) - R(1 - \epsilon)}{2n}$ . This inequality implies

$$\begin{aligned} R(1) - 2nc &> R(1 - \epsilon) \\ &\geq P(v_1), \quad \forall v_1 \leq 1 - \epsilon \end{aligned}$$

that is, the issuer's payoff under  $v_1 \leq 1 - \epsilon$  is less than under  $v_1 = 1$ . In other words, if  $0 < c < \bar{c}$ , the optimal  $v_1 > 1 - \epsilon$ . Because  $p_1$  and  $p_2$  are continuous in  $v_1$ , and  $p_1(1) = E[s^{(n)}(I \setminus i)] > E[s^{(n+1)}(I)] = E[p_2(v_1 = 1)]$ , for small enough  $\epsilon$ , we have  $p_1 > E[p_2]$ .

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