

Culture, Competence, and the Corporation

Alan D. Morrison

Saïd Business School and Merton College,
University of Oxford.

William J. Wilhelm, Jr.

McIntire School of Commerce,
University of Virginia;
Saïd Business School,
University of Oxford.

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Abstract

We provide an economic treatment of two central ideas from management studies: *corporate culture*, and *corporate competence*. We follow Weber and Camerer's (2003) experimental work, which identifies both the importance of cultural norms in communication, and the efficiency costs of moving to an unfamiliar culture. These costs reduce employee mobility and hence serve to incentivise employer-financed training in general skills. To the extent that cultural ties bind skilled employees to the firm, their competences are the corporation's. This suggests a cultural link between technological shocks and training incentives: if new information systems reduce the cultural specificity of communication channels then employees will become more mobile and firms will perform less training. Advances in information technology will therefore increase the demand for professional schools, and will increase employee mobility.

KEY WORDS: Corporate culture, training, information technology.

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Correspondence address: Alan Morrison, Merton College, Oxford OX1 4JD, UK. email: alan.morrison@sbs.ox.ac.uk. Bill Wilhelm, McIntire School of Commerce, Monroe Hall, Charlottesville, VA 22903, USA. email: william.wilhelm@sbs.ox.ac.uk

In this paper we provide an economic framework in which we analyse two ideas which are common in the management literature, but which have received little attention from economists: firstly, the idea that corporations derive value from the possession of *core competencies*; and secondly, the notion of *corporate culture*. We present a theory in which the firm serves as a repository of cultural norms, and we show that the existence of these norms is sufficient to tie skilled personnel to the firm. Our analysis suggests a positive relationship between an organization's cultural differentiation and its incentive to train its staff. We discuss the impact of innovations in communication technologies upon this relationship.

Corporate competencies have been central to the strategic planning literature for over a decade. A company's competencies represent its collective expertise: they were discussed at length by Prahalad and Hamel (1990), who argued that corporations should design strategies in order to extend and to exploit their competencies. Crucial to the notion of a competence is the difficulty of replicating it in other corporations. Prahalad and Hamel's seminal paper spawned a literature examining the implications of their ideas: see for example Collis and Montgomery (1995), and Conner (1991).

The importance of competencies appears to be a central tenet of management strategy. But the idea sits uncomfortably with contractual views of the corporation. People, rather than companies, have skills. One would expect Bertrand competition between potential employers in an industry to ensure that the rent accruing to a particular skill flows to the person in whom the skill resides. For example, in the investment banking world Goldman Sachs have historically been viewed as having unique skills in managing wholesale customer relationships, and the firm has tended to invest heavily in its sales people. Wholesale customers tend to trade standard products which could be supplied by any of the major investment banks: the sales person earns a rent on her knowledge of the specific needs of her client. It is perhaps surprising that, once she has learnt about her customers, the sales person does not simply take her special skills to the highest bidder. And given the danger that this will happen, it is hard to understand why Goldman Sachs should invest so much to endow her with transferrable and non-specific human capital.

One possible explanation of this phenomenon is the difficulties which employees experience in moving from one corporate culture to another. The word "culture" was introduced to the English language by the anthropologist Edward B. Tylor (1871). It is notoriously hard to define: culture is experienced in different ways according to the perspective of the observer (Martin, 1992). Culture

generates shared assumptions and behaviour patterns: it is manifested in organizational artifacts such as modes of access to senior management and the location of the coffee machine. At a deeper level, culture represents deeply-held shared values of the corporation's employees (Schein, 1984).

Management writers have pointed to the importance of organizational culture in organizational success (see for example Peters and Waterman, 1982). At the same time, the depth of cultural assumptions and the difficulty of measuring and describing them renders them hard to change (Barney, 1986). A good organizational culture might therefore be a source of sustained competitive advantage (Wilson and Rosenfeld, 1990).

Although this argument is widely accepted in the management literature it has attracted far less interest from economists. In this paper we will adopt Bower's (1966) early characterization of culture as "the way we do things around here." Specifically, we assume that cultural norms determine the way that information is communicated within the firm.

Our identification of cultural norms with the organization's informal communication channels is consistent with much of the small economic literature on culture. Arrow (1974) suggests that organizational culture is constituted of codes developed within organisations to coordinate activities. Crémer (1993) thinks of culture as a store of common knowledge and language upon which the employees can draw to save costly time when responding to external stimuli. Further discussion of optimal organizational codes is provided by Crémer, Garicano and Prat (2004). Lazear (1999) notes that cultural assimilation facilitates communication and hence facilitates trade between individuals.

Some experimental evidence of the importance of shared norms and experience in organizational communication is provided by Weber and Camerer (2003). They gave one subject 8 from 16 pictures and they recorded the time he took to give another subject sufficient information to enable her to select the same pictures in the same order. Over 20 rounds of the same game, the time taken to perform this task dropped an average of 249 seconds to 48, as the subjects found verbal shortcuts for describing the pictures. Merging teams so that one member was unfamiliar with the private language developed by the other two raised task completion times to 130 seconds, after which convergence to pre-merger competence levels was extremely slow.

In line with Weber and Camerer, we consider a model in which employees acquire *cultural skills* in the early part of their careers without which they cannot operate efficiently. As discussed above, culture is a set of learned tacit skills which facilitate communication within the organization. For example, the sales person may learn how to use a set of organization-specific artifacts to

communicate her activities to her supervisor. In addition to verbal and written reports these may include an understanding of the formal and informal lines of reporting in the organization, and an understanding of the most effective ways to attract attention and support when needed. We further follow Weber and Camerer by assuming that an employee who leaves the firm after acquiring cultural skills experiences cultural dissonance which impairs his efficiency.

We depart from Cremer's (1993) model and the experimental setting employed by Weber and Camerer (2003) by considering a model of the firm in which there are agency problems between the manager and the employees. As in Alchian and Demsetz (1972), the role of the manager is to monitor the actions of the firm's employees and hence to ensure that they act in a value-maximising way. Monitoring is performed using both formal and informal reporting and communication lines which rely upon cultural skills. An employee who is not versed in the organization's culture is unable properly to communicate with his managers and so cannot be monitored as effectively. He therefore cannot be granted much autonomy of action and hence the value to the firm of his skills is reduced.

We analyse in this setting the interaction between cultural skills and transferable *functional skills*. Functional skill is part of an agent's general human capital and it is perfectly transferable, in the sense that it will be equally as productive if deployed in an organization which can provide the necessary infrastructural support. In the context of our investment banking example, any agent who sells interest rate swaps can equally use her skills at any of a number of investment banks.

Functional skill can be acquired either through professional training paid for by the employee, for example on an MBA programme, or it can be paid for by the employer through mentoring in the early stages of the career. Since an employee's productivity is adversely affected by the cultural displacement which he experiences upon leaving the firm, the employer will earn quasi-rents from his functional skill. If these quasi-rents are sufficiently high then the employer will be prepared to pay for training: if not the employee will purchase it himself. Corporate investment in general skills and the accumulation of firm-specific competencies are therefore susceptible to a cultural explanation.

Our analysis suggests the existence of a cultural channel which links communications technology to employee training and mobility. Advances in information processing which codify previously tacit communications systems will serve to undermine the role of organization-specific cultural norms in monitoring employees. This will reduce the efficiency cost of changing firms and so will increase

the return which an employee earns on his functional skills. This will undermine the employer's incentives to train its staff and so will result in an increased demand for professional qualifications.

The investment banking industry again provides an example of this phenomenon. The introduction of risk management techniques based around Value at Risk (VaR) reports have recently revolutionized trading businesses. VaR reports provide simple and standardized statistics about risk-taking which allow managers easily to quantify the risks which their employees are taking. Since its introduction in the early 1990s in J. P. Morgan, VaR reporting has become ubiquitous. It has provided standard codes for communicating risk information within the firm and has replaced earlier approaches which relied to a large extent upon informal communication and trust. As VaR has replaced tacit with codifiable communications channels it has increased employee mobility and has reduced the rent which corporations earn from their trading activities. As a result, one would expect employees to bear a greater proportion of their training costs. Casual empiricism confirms that this is the case.

Although we believe that our model captures an essential quality of corporate culture, it leaves some aspects unaddressed. Much of the management literature addresses the behavioural characteristics of organizational culture. Kreps (1990) argues that culture can enable corporate actors to decide between multiple equilibria, and shows how reputational incentives can sustain cultural norms. Hermalin's (1991) survey expands upon this theme. A related question is the extent to which a productive culture can be nurtured: this is addressed by Rob and Zemsky (2002) in a model in which workers reciprocate the behaviour which they experience from their peers.

Cultural assimilation is left unmodelled in our paper. Carillo and Gromb (1999, 2002) address this question by examining worker incentives to make culture-specific investments. Such investments generate positive externalities for other workers as they increase the firm's costs of cultural change. The interesting problem of managerial myopia when assessing local culture and the obstacles it places in the way of cultural change is currently unexamined in the economics literature.

We formalise our discussion in sections 1 and 2 with a simple model of a two firm economy. Section 3 examines the relationship between culture and investment in training. Section 4 contains a discussion of our results and suggests some extensions to the model. Section 5 concludes.

1. Model

We consider the interaction between two firms and an employee. The action unfolds over two periods: period 1 runs from time 0 to time 1 and period 2 runs from time 1 to time 2.

We are concerned in this model with the incentives for general human capital production and their interaction with the operation of the labour market. We refer to general human capital as *functional skill*. The cost of acquiring functional skill λ is $C(\lambda)$, where $C'(\cdot) > 0$ and $C''(\cdot) > 0$. At time 0 the employee has no functional skills but he can if he wishes purchase training before starting work. We write λ^e for the level of functional skill which he purchases.

Employees cannot be productive without both functional skills and *product knowledge*. We think of product knowledge as essentially tacit: it can be acquired only through on-the-job experience and not in a classroom. Product knowledge acquired in one firm is transferable to the other one. An example from investment banking is detailed knowledge of a specific client's needs and preferences. Sales people who learn about a client frequently take the client with them if they move from one investment bank to another.

At time 0 the employee enters the labour market. He acquires product knowledge by working in period 1, but while he does so he is unproductive.¹ During period 1 the employer can elect if it chooses to pay for additional training which will raise the employee's functional skill level to $\lambda \equiv \lambda^e + \lambda^f$. At the end of the period the employee re-enters the labour market and in period 2 he uses his functional skills productively.

The nature of the employee's interaction with his second period employer is determined by his *cultural fit* κ with the firm. At time 0 the employee has no cultural skills, but he acquires them during period 1. At time 1, his fit with the period 1 employer is high ($\kappa = \kappa_h$), and with the competitor is low ($\kappa = \kappa_l > \kappa_h$). This reflects the discussion in the introduction of Weber and Camerer's (2003) experimental findings. Cultural fit does not affect the employee's productivity and, unlike product knowledge, cultural skills are non-transferrable.

In period 2 the employee can use his functional skill and product knowledge to work at a productive task. The task requires an initial investment of K and will return R or 0. The employee selects an effort level $e \in \mathfrak{R}_{\geq 0}$ at cost $e^2/2$. The probability that the task returns R is $\pi(e, \lambda)$. We

¹This assumption is intended to ensure that an inexperienced employee is not particularly valuable and is without significant loss of generality. It would be a simple matter to modify the model to allow the employee to perform unskilled clerical work in the first period.

assume that π is twice continuously differentiable, increasing in e and λ , and concave in (e, λ) .

We assume that there is an agency problem between the employer and the employee. If the employee is not adequately monitored then he will appropriate the returns from a successful project with sufficient probability to render investment unattractive to the employer. Appropriation is unobservable and impairs the ability of the employee and the employer to contract. This problem can be overcome by establishing adequate channels of communication *ex ante*, so that the employer can monitor the employee and prevent appropriation. These channels depend upon the employee's ability to use the firm's formal and informal lines of reporting. In other words, as we discuss in the introduction, they rely upon the cultural fit between the employee and the employer. Specifically, we assume that the probability of establishing adequate reporting lines and hence of overcoming the agency problem through monitoring is equal to the cultural fit parameter κ .

If the appropriate monitoring systems are not established then the employer anticipates appropriation and hence refuses to invest. When investment occurs, the employee is (optimally) paid only when the task succeeds. We write w for the wage payment in the event that that task returns R .

It follows from this discussion that the second period income of an employee with functional skill λ working in a firm with cultural fit κ and exerting effort e in exchange for incentive wage w is:

$$I(\kappa, w, \lambda, e) \equiv \kappa w \pi(e, \lambda) - e^2/2. \quad (1)$$

The profit which a firm generates from this employee is as follows:

$$P(\kappa, w, \lambda, e) \equiv \kappa(R - w) \pi(e, \lambda) - K. \quad (2)$$

In the following sections we solve this model by backwards induction to determine the personal and firm-level investments λ_e and λ_f in human capital.

2. Second Period Decisions

Given cultural fit κ , employment contract w , and functional skill λ , the optimal period 2 effort level $\bar{e}(\kappa, w, \lambda)$ satisfies the following first order condition:²

$$\bar{e} = \kappa w \pi_1(\bar{e}, \lambda). \quad (3)$$

²To avoid notational clutter, throughout the paper we use subscripts to denote partial derivatives: hence $\pi_1(e, \lambda) = \frac{\partial \pi}{\partial e}(e, \lambda)$, $\pi_{11}(e, \lambda) = \frac{\partial^2 \pi}{\partial e^2}(e, \lambda)$ and so on.

Let $\bar{I}(\kappa, w, \lambda) \equiv I(\kappa, w, \lambda, \bar{e}(\kappa, w, \lambda))$ be the income level corresponding to $\bar{e}(\kappa, w, \lambda)$. Define $w^h(\kappa_h, \kappa_l, w, \lambda)$ to be the wage which an employer with high cultural fit (κ_h) has to pay to give an employee with functional skill λ the expected income which he would derive from a wage w paid by an employer with low cultural fit (κ_l):

$$\bar{I}(\kappa_h, w^h(\kappa_h, \kappa_l, w, \lambda), \lambda) = \bar{I}(\kappa_l, w, \lambda). \quad (4)$$

LEMMA 1 $w^h(\kappa_h, \kappa_l, w, \lambda) = \kappa_l w / \kappa_h < w$.

Lemma 1, whose proof appears in the appendix, is intuitively obvious. An employee who exerts a high level of functional effort will be rewarded for doing so only if his employer is able to understand his project. So a poor cultural fit reduces the responsiveness of income to effort and so undermines the effectiveness of incentive pay. As a result, employers with a strong cultural fit with their employees can pay them less.

Define

$$w^*(\kappa_h, \kappa_l, \lambda) \equiv w_h(\kappa_h, \kappa_l, R, \lambda); \quad (5)$$

$$e^*(\kappa_h, \kappa_l, \lambda) \equiv \bar{e}(\kappa_h, w^*(\kappa_h, \kappa_l, \lambda), \lambda). \quad (6)$$

PROPOSITION 1 *The employee remains with the same employer in the second period and is paid $w^*(\kappa_h, \kappa_l, \lambda)$ provided condition (7) is true:*

$$\frac{\kappa_h (R - w^*(\kappa_h, \kappa_l, \lambda))}{1 - \kappa_h w^* \pi_{11}(e^*(\kappa_h, \kappa_l, \lambda), \lambda)} \frac{[\pi_1(e^*(\kappa_h, \kappa_l, \lambda), \lambda)]^2}{\pi(e^*(\kappa_h, \kappa_l, \lambda), \lambda)} < 1. \quad (7)$$

When condition (7) is false the employee is paid the wage $\hat{w} > w^(\kappa_h, \kappa_l, \lambda)$ which makes the condition hold with equality.*

The proof of proposition 1 appears in the appendix. It follows intuitively because the highest payment which the competitor can afford to make is R and an offer of $w^*(\kappa_h, \kappa_l, \lambda)$ from the existing employer is equally attractive to the employee. The employer experiences a direct cost when wages are increased, and an indirect gain from the higher effort which they induce. Condition (7) is satisfied when the former effect outweighs the latter. This occurs when κ_l is sufficiently high so that competitive pressures allow the employee to extract a significant rent from his functional skills. We assume for the remainder of the paper that this is the case.

Proposition 1 states that the employee will not change firms after the first period of his career. This is true because his existing employer's superior cultural skills give it an advantage in resolving

the principal/agent problem which exists between them. The existing employer can therefore match any outside option, and still retain some of the profits from the employee's actions. In other words, functional skills which reside in the corporation after one period will remain there.

Our model therefore provides an explanation for the creation of "corporate competencies:" although these are general skills which reside within the employees of the firm, they are tied to the firm by cultural norms which are valuable in aligning the employee's incentives with those of the employer. This remark applies equally to the retention within the firm of product knowledge. A unique corporate culture allows the firm to establish property rights over key assets such as client relationships. It may help to explain the existence of organizations such as investment banks, which exist essentially to invest in and to maintain informational assets whose ownership cannot be established in court.

We make the following definitions:

$$I^*(\kappa_h, \kappa_l, \lambda) \equiv \bar{I}(\kappa_h, w^*(\kappa_h, \kappa_l, \lambda), \lambda); \quad (8)$$

$$P^*(\kappa_h, \kappa_l, \lambda) \equiv P(\kappa_h, e^*(\kappa_h, \kappa_l, \lambda), w^*(\kappa_h, \kappa_l, \lambda), \lambda); \quad (9)$$

$$\pi^*(\kappa_h, \kappa_l, \lambda) \equiv \pi(e^*(\kappa_h, \kappa_l, \lambda), \lambda). \quad (10)$$

With assumption (7), w^* , e^* , I^* , P^* , and π^* are respectively the equilibrium period 2 wage, effort level, employee income, employer profits and probability of task success. Define $\mathcal{W}^*(\kappa_h, \kappa_l, \lambda)$ as follows:

$$\mathcal{W}^*(\kappa_h, \kappa_l, \lambda) \equiv I^*(\kappa_h, \kappa_l, \lambda) + P^*(\kappa_h, \kappa_l, \lambda). \quad (11)$$

\mathcal{W}^* is the total expected second period surplus after investments in the employee's functional skill λ have been performed and hence is a measure of *ex post* welfare.

The following proposition examines comparative statics with respect to κ_h and κ_l .

PROPOSITION 2

1. e^* , π^* and I^* are unaffected by changes in κ_h and are increasing in κ_l ;
2. P^* is increasing in $\Delta\kappa$;
3. \mathcal{W}^* is increasing in κ_l and in κ_h .

The intuition for these results is as follows. Firstly, recall that the employee's period 2 income is equal to his outside option. This is unaffected by κ_h , and so nor is his effort or income. Since

the probability of project success depends upon employee effort this is also unaffected by κ_h and is increasing in κ_l .

Part 2 of the proposition is best illustrated by setting $\kappa_h w^*(\kappa_h, \kappa_l, \lambda) = \kappa_l R$ in the expression for P^* to yield the following expression:

$$P^*(\kappa_h, \kappa_l, \lambda) = R\Delta\kappa\pi^*(\kappa_h, \kappa_l, \lambda) - K, \quad (12)$$

where $\Delta\kappa \equiv \kappa_h - \kappa_l$ is a measure of the *cultural differentiation* between the two firms. Cultural differentiation imposes a productivity cost upon employees who switch firms at time 1 and this cost is extracted as period 2 rent by the initial employer. Intuitively therefore, we expect greater cultural differentiation to raise employer profits. Note though that changes in $\Delta\kappa$ have an indirect effect upon π^* so that we need to check that this effect does not outweigh their direct impact upon P^* . This is accomplished in the appendix.

Finally, increases in κ_h do not affect the probability of project success, but they do affect the probability of project execution and so raise *ex post* welfare. This increase accrues entirely to the period 1 employer without affecting the employee's wealth. Increases in κ_l raise the probability of project success without changing the probability of project execution and hence raise *ex post* welfare. This increase flows entirely to the employee, who by part 2 of the proposition will in this case also receive a wealth transfer from the employer.

We conclude this section by examining the comparative statics with respect to functional skill (λ).

PROPOSITION 3

1. w^* is unaffected by λ , while I^* is increasing in λ ;
2. $e^*(\kappa_h, \kappa_l, \lambda)$ is increasing in λ if and only if $\pi_{12}(e^*, \lambda) > 0$;
3. π^* and P^* are all increasing in λ if and only if

$$e_3^*(\kappa_h, \kappa_l, \lambda) > -\frac{\pi_2(e^*, \lambda)}{\pi_1(e^*, \lambda)}, \quad (13)$$

which is true if and only if

$$\pi_{12}(e^*, \lambda) > -\frac{1 - \kappa_h w^*(\kappa_h, \kappa_l, \lambda) \pi_{11}(e^*, \lambda)}{\kappa_h w^*(\kappa_h, \kappa_l, \lambda)} \frac{\pi_2(e^*, \lambda)}{\pi_1(e^*, \lambda)}, \quad (14)$$

4. \mathcal{W}^* is increasing in λ if and only if

$$e_3^*(\kappa_h, \kappa_l, \lambda) > -\frac{\pi_2(e^*, \lambda)}{\pi_1(e^*, \lambda)} \left(1 + \frac{\kappa_h w^*(\kappa_h, \kappa_l, \lambda)}{R\Delta\kappa} \right),$$

which is true if and only if

$$\pi_{12}(e^*, \lambda) > -\frac{1 - \kappa_h w^*(\kappa_h, \kappa_l, \lambda) \pi_{11}(e^*, \lambda) \pi_2(e^*, \lambda)}{\kappa_h w^*(\kappa_h, \kappa_l, \lambda) \pi_1(e^*, \lambda)} \left(1 + \frac{\kappa_h w^*(\kappa_h, \kappa_l, \lambda)}{R\Delta\kappa}\right).$$

At first blush, it may appear surprising that $\partial w^*/\partial\lambda = 0$ so that an increase in functional skill does not earn the employee a higher incentive wage. The reason for this is that increasing functional skill raises the value of the outside option, but it also raises the value of staying at the existing employer. The employer need only compensate the employee to the extent that the former effect outweighs the latter. In our set-up, these effects cancel one another out. Note though that increased functional skill raises the value of projects which receive the go-ahead from the existing employer and hence that it increases the employee's expected income: $\partial I^*/\partial\lambda > 0$.

Intuitively, increased functional skill will cause the employee to work harder only when skill and effort are complements: i.e., when $\pi_{12} > 0$. When $\pi_{12} < 0$ so that effort substitutes for skill, increased functional skill will serve to lower the employee's effort.

Condition (13) states that increased skill will raise welfare (and profits and the success probability) when it serves to increase effort further than would be required to remain on the same iso- π curve. This is equivalent (equation 14) to the statement that effort and skill are not too substitutable: when they are, the employee's gains are partially at the expense of the employer. The final part of the proposition shows that for sufficiently low $e_3^*(\kappa_h, \kappa_l, \lambda)$, the employee gains from higher λ are entirely at the employer's expense, and that *ex post* welfare is reduced.

3. Investment in Functional Skills

In this section we analyze the employee's time 0 and the employer's period 1 equilibrium investments in functional skill, which we denote by $\lambda^e(\kappa_h, \kappa_l)$ and $\lambda^f(\kappa_h, \kappa_l)$ respectively. We write

$$\lambda(\kappa_h, \kappa_l) \equiv \lambda^e(\kappa_h, \kappa_l) + \lambda^f(\kappa_h, \kappa_l)$$

for the total investment in human capital. Firstly, we establish the following result for the marginal returns to skill for the employee and the employer:

LEMMA 2

1. $I_{33}^*(\kappa_h, \kappa_l, \lambda) < 0$;
2. $P_{33}^*(\kappa_h, \kappa_l, \lambda) < 0$ if and only if condition (15) is satisfied:

$$e_{33}^*(\kappa_h, \kappa_l, \lambda) < -\frac{1}{\pi_1(e^*, \lambda)} \left\{ \pi_{11}(e^*, \lambda) (e_3^*)^2 + 2\pi_{12}(e^*, \lambda) e_3^* + \pi_{22}(e^*, \lambda) \right\}, \quad (15)$$

where e^* and its derivatives are evaluated at $(\kappa_h, \kappa_l, \lambda)$.

Note that the curly-bracketed term in condition (15) is negative by virtue of the concavity of π . The condition therefore requires e_{33}^* to be small and positive, or negative. This is ultimately a statement about the third derivatives of π : we adopt it as an assumption.³

We define $L^e(\kappa_h, \kappa_l)$ and $L^f(\kappa_h, \kappa_l)$ as follows:

$$I_3^*(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l)) = C'(L^e(\kappa_h, \kappa_l)); \quad (16)$$

$$P_3^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l)) = C'(L^f(\kappa_h, \kappa_l)), \quad (17)$$

and we assume that

$$I^*(\kappa_l, \kappa_l, L^e(\kappa_l, \kappa_l)) - C(L^e(\kappa_l, \kappa_l)) > 0. \quad (18)$$

Conditional upon no investment by the employer, $L^e(\kappa_h, \kappa_l)$ is the investment which the employee makes in functional skill (λ); assumption (18) states that this level of investment is individually rational for the employee when $\kappa_h = \kappa_l$. Since I^* is κ_h -invariant, the employee's participation constraint in the absence of employer investment is satisfied for all κ_h .

If $P_3^*(\kappa_h, \kappa_l, \lambda)$ was negative then the employer would never invest in functional skill. To rule out this uninteresting case, we assume that condition (13) is satisfied. Then, subject to satisfying his participation constraint, the employer would be prepared to invest in functional skill precisely until the total level of skill was $L^f(\kappa_h, \kappa_l)$.

LEMMA 3 *In equilibrium, either:*

1. $\lambda^e(\kappa_h, \kappa_l) = L^e(\kappa_h, \kappa_l)$ and $\lambda^f(\kappa_h, \kappa_l) = 0$;
2. or $\lambda^e(\kappa_h, \kappa_l) = 0$ and $\lambda^f(\kappa_h, \kappa_l) = L^f(\kappa_h, \kappa_l)$.

Proof. If $\lambda^f(\kappa_h, \kappa_l) = 0$ then by definition $\lambda^e(\kappa_h, \kappa_l) = L^e(\kappa_h, \kappa_l)$. If $\lambda^f(\kappa_h, \kappa_l) > 0$ then again by definition, $\lambda^f(\kappa_h, \kappa_l) = L^f(\kappa_h, \kappa_l)$. In this case an employee who invested in functional skills would therefore bear some of the costs of training without changing period 2 functional skills: employee investment is valuable only insofar as it generates a period 1 return. But investment in the employee's skill does not bear fruit until after period 1 on-the-job learning. Hence $L^e(\kappa_h, \kappa_l) = 0$.

□

³For example, it is easy to see that the assumption will be true whenever $\pi(e, \lambda)$ takes the form $f(e) + g(\lambda) + k(e)l(\lambda)$ for concave f , g , k , and l , with f''' and k''' both negative, or small and positive.

We write E^e and E^f for the employee's expected income in cases (1) and (2) of lemma 3, and F^e , F^f for the firm's income:

$$\begin{aligned} E^e(\kappa_h, \kappa_l) &\equiv I^*(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l)) - C(L^e(\kappa_h, \kappa_l)); \\ E^f(\kappa_h, \kappa_l) &\equiv I^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l)); \\ F^e(\kappa_h, \kappa_l) &\equiv P^*(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l)); \\ F^f(\kappa_h, \kappa_l) &\equiv P^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l)) - C(L^f(\kappa_h, \kappa_l)). \end{aligned}$$

The IR constraint (18) therefore reduces to $E^e(\kappa_h, \kappa_l) > 0$ and lemma 3 shows that the training IR constraint for the employer is $F^f(\kappa_h, \kappa_l) > 0$. In the case where this not the case it is convenient to define $L^f(\kappa_h, \kappa_l) = 0$. With this definition it is obvious that $\lambda^e(\kappa_h, \kappa_l) = L^e(\kappa_h, \kappa_l)$ precisely when $E^e(\kappa_h, \kappa_l) \geq E^f(\kappa_h, \kappa_l)$. We now examine the effect of the cultural parameters κ_h , κ_l upon incentives to invest in functional skills:

LEMMA 4

1. L^e is unaffected by κ_h and is increasing in κ_l ;
2. L^f is increasing in κ_h . It is decreasing in κ_l precisely when condition (19) is satisfied:

$$\Delta\kappa\pi_{23}^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l)) < \pi_3^*(\kappa_h, \kappa_l). \quad (19)$$

The intuition behind part 1 of lemma 4 is straightforward. Since changes in κ_h do not affect the employee's outside option they do not affect his incentives to invest in functional skills; increases in κ_l raise the return which he can extract from his human capital and so incentivize its augmentation.

The first statement in part 2 is intuitively obvious: since the employer captures all of the efficiency gains from higher κ_h , we must have $\partial L^f / \partial \kappa_h > 0$. The second statement highlights two effects which a higher κ_l has upon the employer's incentives. Firstly, there is a direct effect represented by the term on the right hand side of equation (19): *ceteris paribus*, higher κ_h reduces the share of the returns to functional skill which the employer can capture and hence reduces his incentive to invest in functional skill. Secondly, there is an indirect effect represented by the term on the left hand side of the equation. The marginal effect of λ may be altered by changes in κ_l and this may affect investment incentives either positively or negatively, according to the sign of π_{23}^* .

Further expansion of expression (19) is not illuminating. For the remainder of the paper we will assume that the direct effects of increases in κ_l outweigh the indirect effects and hence we

adopt equation (19) as an assumption. Like equation (15), it is satisfied for a number of plausible functional forms for π .

Let

$$\mathcal{W}(\kappa_h, \kappa_l) \equiv \mathcal{W}^*(\kappa_h, \kappa_l, \lambda(\kappa_h, \kappa_l)) - C(\lambda(\kappa_h, \kappa_l))$$

be the *ex ante* expected welfare: in case (1) of lemma 3 we have $\mathcal{W} = E^e + F^e$, and in case 2, $\mathcal{W} = E^f + F^f$.

PROPOSITION 4 *There exists $\bar{\kappa}_h(\kappa_l) \leq \infty$, with $\bar{\kappa}_h'(\kappa_l) > 0$, such that $\lambda^e(\kappa_h, \kappa_l) = L^e(\kappa_h, \kappa_l)$ precisely when $\kappa_h \leq \bar{\kappa}_h(\kappa_l)$. Expected welfare $\mathcal{W}(\kappa_h, \kappa_l)$ is continuously increasing in κ_h for $\kappa_h \neq \bar{\kappa}_h(\kappa_l)$, and exhibits a discontinuous drop at $\kappa_h = \bar{\kappa}_h(\kappa_l)$.*

Proposition 4 is the natural consequence of our formalization of corporate culture. When cultural differences serve to reduce labour mobility, functional skills are valuable to the employee only insofar as they would affect his productivity at competitor firms. When the cultural gulf between firms is so great that the employee would find it hard to employ his functional skills anywhere else, the employer will have a greater incentive to invest in training than the employee, and the employee will no longer pay for his own education.

Note that increases in κ_l serve to increase employee's ability to communicate with other employers and hence raise the marginal value which he places upon functional skill. The range of κ_h values for which the employee educates himself is therefore increasing in κ_l .

We show in the proof of proposition 4 that E^e is unaffected by κ_h and that E^f , F^e and F^f are all increasing in κ_h away from $\bar{\kappa}_h$. Note that the employee decides whether or not to invest in functional skills. At the critical value $\bar{\kappa}_h(\kappa_l)$, $E^e(\kappa_h, \kappa_l) = E^f(\kappa_h, \kappa_l)$ and he is indifferent between training himself and relying upon training provided by his employer. We therefore have:

COROLLARY 1 *The employee's welfare is a continuous weakly increasing function of κ_h . The employer's welfare is a continuous increasing function of κ_h away from $\bar{\kappa}_h(\kappa_l)$, and the employer experiences the whole of the welfare drop at $\bar{\kappa}_h(\kappa_l)$.*

4. Discussion

4.1. *Technological Shocks in Professional Services Firms*

We argue that our theory is of particular relevance in the context of professional services firms such as law partnerships and investment banks. Professional services firms sell skills which are embedded in their employees. Employees have traditionally received much of their functional training on-the-job. Junior employees are typically cash-constrained and hence, while they may to some extent pay for their training as they receive it through lower wages or longer working hours, some payment must occur after the training has occurred. In a world of imperfect contracts and inalienable human capital, post-education payment is limited by the size of the employee's outside option.

Our model provides a positive role in this context for cultural differences: when skills are acquired on the job, the difficulty of adapting to a new corporate culture restricts the employee's mobility and hence guarantees the employer a return on his investment in training. It is clear from our model that, in the case where the employee is unable to acquire functional skills without on the job training, cultural differences are essential to their dissemination.

Notwithstanding these observations, there has been in recent years a significant drop in workplace training in professional services. For example, Hillman (2001) documents a sharp rise in lawyer mobility in the United States. Coincident with this rise has been a sharp reduction in training activities: Hillman states that

“Mentoring is haphazard, if it exists at all. Firms point to the new economics of law practice and ask law schools to do more.” (*ibid*, 2001, p. 1078).

This observation would be consistent in our model with an increase in κ_l : according to proposition 4, this will increase the range of cultural parameters κ_h for which employees will purchase their own training. Similarly, the last two decades have seen a sharp increase in the demand for M.B.A. graduates in investment banking. The increased reliance upon self-training in investment banking is again explicable in terms of an increase in the ambient culture parameter, κ_l .

One possible explanation for increases κ_l values in professional services firms is recent advances in information technology. The advent of the personal computer and the development of intelligent software has increased the codifiability of functional skill. At the same time, it has standardized much of the reporting which occurs within professional services firms. For example, the advent

in the investment banking world of standard risk management systems such as J.P. Morgan's *RiskMetrics* has given investment banks the tools they need to monitor new hires and to ensure that they are immediately productive. In short, we contend that technological advantages have raised the base quality of within-firm communication and hence have reduced the importance of firm-specific cultural artifacts in facilitating new business. As a result human capital is more mobile and more training is purchased outside the firm.

4.2. Employee Mobility

The increased emphasis upon professional training identified above in the law and investment banking industries has occurred at the same time as a marked increase in employee mobility. In our simple model, employees never switch firms: this feature clearly does not fit the real world. Introducing such a feature would however be a simple matter, although it would significantly increase the complexity of the model without adding any further insights.

One natural way to incorporate labour mobility would be to endow each company with a different production technology, with the stochastic return R on successful projects drawn from a common distribution before the opening of the time 1 labour market. Employees would continue to work in period 2 for the firm at which their functional skills would be most useful. As the two companies' returns would be i.i.d., the most likely period two employer would be the one with the highest cultural fit, but with non-zero probability the competitor's returns would exceed those of the incumbent employer by a sufficient amount to compensate for its lower cultural fit. Naturally, as $\Delta\kappa$ decreased, the probability of this event would increase and greater employee mobility would occur simultaneously with an increased emphasis upon self-training. Moreover, in this extension anticipated labour mobility would serve further to reduce the incumbent employer's incentives to invest in employee skills.

4.3. Team Transfers

We believe that our model may go some way towards explaining the investment banking phenomenon of "team transfers" under which an entire division move from one firm to another, with even the most junior members of the division receiving a substantial salary hike. If the cultural norms of the team are hard to learn and they are the source of the employing firm's rent, a team which

can take its culture with it will hugely increase its value. Insofar as the culture is embedded in junior staff one would expect them to receive a share of its value.

4.4. *Optimal Cultural Differentiation*

In our model cultural differences tie employees to the corporation and hence allow the company to acquire competencies. We therefore provide an explanation for the assertion, common in the management studies literature, that a unique organizational culture can give organizations a competitive edge (Wilson and Rosenfeld, 1990). This observation has generated a related literature which discusses the extent to which corporations can “manage” their cultures (Peters and Waterman, 1982).

Most academic writers have concluded that active cultural management is extremely difficult: cultural norms are so deeply embedded in the groups which manifest them that they are hard to measure or even to quantify. Our model suggests that, far from representing a problem to the corporation, this feature of culture is central to its value. To the extent that culture can be codified, it can be replicated. Replication of a successful culture will allow employees to move between corporations without experiencing the costs of cultural dissonance: in the context of our model, it will raise κ_l . Cultural differences are valuable to the corporation *precisely because* they cannot be codified.

Nevertheless, we can use our model to think about cultural management. Active cultural management will in our set-up result in an increase in κ_h . We know from corollary 1 that this will raise employer welfare everywhere except at $\bar{\kappa}_h(\kappa_l)$, where it will cause a discontinuous drop in welfare. For fixed κ_l an employer with control over cultural norms will therefore elect to raise κ_h either to $\bar{\kappa}_h - \varepsilon$, or will raise them sufficiently far above $\bar{\kappa}_h$ to cover the welfare loss which he experiences as he starts to train. Provided the costs of cultural differentiation are not too severe, for given κ_l one would therefore expect corporations to differentiate themselves as far as possible along cultural lines.

In practice, the extent to which this form of differentiation is possible depends upon the number of companies and the topology of the κ space. For example, suppose that cultural values are located on a circle of radius ρ , and that cultural differentiation is measured along its circumference. Then with n firms the maximum degree of differentiation will be $2\pi\rho/n$: if this is just in excess of $\bar{\kappa}_h$

then firms will optimally cluster. A detailed model of this type of effect would allow us to examine the effects of cultural differences upon entry incentives, and is outside the scope of this paper.

The effect of an increase in κ_l upon the employer's preferred $\Delta\kappa$ is in our model ambiguous. Since $\bar{\kappa}'_h(\kappa_l) > 0$, firms with $\Delta\kappa = \bar{\kappa}_h - \varepsilon$ might be expected to react by attempting to raise κ_h . On the other hand, firms with κ_h slightly above $\bar{\kappa}_h$ might find it cheaper to reduce cultural specificity to below the new $\bar{\kappa}_h$ than to raise it still further. To the extent that increases in κ_l presage increased costs of cultural management, one might expect the latter effect to obtain. In any case, we have argued above that cultural tweaking of the type discussed in this paragraph is probably impossible.

5. Conclusion

In this paper we examine the consequences for general human capital production and mobility of corporate culture. We view corporate culture as comprising tacit firm-level communication skills which are essential to managerial monitoring. An employee's general human capital is productive only insofar as he can be monitored, and the cultural dissonance which he will experience upon leaving his firm therefore impairs his ability to sell his skills elsewhere. This observation explains firm investment in general human capital.

Cultural norms are important in resolving agency problems only insofar as communication technologies are tacit. We argue that advances in information processing and reporting have in recent years facilitated the codification of many formerly tacit channels of reporting. This has increased employee mobility and has disincentivised on-the-job transfer of general skills. As a result we have in recent years witnessed an increasing demand for professional degrees such as MBAs.

We argue that corporate competences arise when employees and their general human capital are culturally bound to their organizations. This generates a new insight into the competence-based view of corporate strategy. When cultural differentiation enables corporations to generate quasi-rents from competences, it seems likely that corporations should enter businesses which reinforce and which rely upon their cultural differences. Hence, corporate strategy is to a large extent associated with cultural management. We leave this question for later research.

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Appendix

Proof of Lemma 1

Applying the Envelope Theorem to equation (1) yields the following

$$\bar{I}_1(\kappa, w, \lambda) = w\pi(\bar{e}(\kappa, w, \lambda), \lambda); \quad (20)$$

$$\bar{I}_2(\kappa, w, \lambda) = \kappa\pi(\bar{e}(\kappa, w, \lambda), \lambda). \quad (21)$$

Differentiate equation (4) with respect to κ_h to obtain

$$w_1^h(\kappa_h, \kappa_l, w, \lambda) = -\frac{\bar{I}_1(\kappa_h, w^h, \lambda)}{\bar{I}_2(\kappa_h, w^h, \lambda)} = -\frac{w^h(\kappa_h, \kappa_l, w, \lambda)}{\kappa_h}. \quad (22)$$

Now define $G(\kappa_h, \kappa_l, w, \lambda) \equiv \kappa_h w^h(\kappa_h, \kappa_l, w, \lambda)$. Then $G(\kappa_l, \kappa_l, w, \lambda) = \kappa_l w$ and $G_1(\kappa_h, \kappa_l, w, \lambda) = w^h(\kappa_h, \kappa_l, w, \lambda) + \kappa_h w^h(\kappa_h, \kappa_l, w, \lambda) = 0$, so that $\kappa_h w^h(\kappa_h, \kappa_l, w, \lambda) = \kappa_l w$ as required.

Note that the right hand side of equation (22) is the slope of iso-income curves in $\kappa_h - w^h$ space: the equation simply states that w^h will be altered in response to changes in κ_h in order to maintain the income of an employee with cultural fit κ_h at $I(\kappa_l, w, \lambda, \bar{e}(\kappa_l, w, \lambda))$. Such an employee has effort level is $\bar{e}(\kappa_h, w^h(\kappa_h, \kappa_l, w, \lambda), \lambda)$: examination of the expression for I therefore suggests that we must have $\bar{e}(\kappa_h, w^h(\kappa_h, \kappa_l, w, \lambda), \lambda) = e(\kappa_l, w, \lambda)$. This is indeed the case; it is demonstrated in the special case where $w = R$ in the proof of proposition 2 below.

Proof of Proposition 1

Write $\tilde{P}(\kappa, w, \lambda) \equiv P(\kappa, \bar{e}(\kappa, w, \lambda), w, \lambda)$ for the profits earned by an employer with cultural fit κ paying incentive wage w to an employee with functional skill λ .

Trivially, no wage w below $w^*(\kappa_h, \kappa_l, \lambda)$ can be an equilibrium since whichever employer pays it, the competitor can afford a wage at which the employee earns a higher income. The incumbent can always attract the employee by paying $w^*(\kappa_h, \kappa_l, \lambda)$. He will elect to do so precisely when $\tilde{P}_2(\kappa_h, w^*(\kappa_h, \kappa_l, \lambda), \lambda) < 0$; he will otherwise select the wage $\hat{w} > w^*$ at which $\tilde{P}_2(\kappa_h, \hat{w}, \lambda) = 0$. Direct differentiation of the definition of \tilde{P} yields

$$\begin{aligned} \tilde{P}_2(\kappa, w, \lambda) &= P_2(\kappa, \bar{e}(\kappa, w, \lambda), w, \lambda) \bar{e}_2(\kappa, w, \lambda) + P_3(\kappa, \bar{e}(\kappa, w, \lambda), w, \lambda) \\ &= \kappa \pi(\bar{e}(\kappa, w, \lambda), \lambda) \left\{ \frac{\kappa(R-w)}{1 - \kappa w \pi_{11}(\bar{e}(\kappa, w, \lambda), \lambda)} \frac{[\pi_1(\bar{e}(\kappa, w, \lambda), \lambda)]^2}{\pi(\bar{e}(\kappa, w, \lambda), \lambda)} - 1 \right\}. \end{aligned}$$

The result is then immediate.

Proof of Proposition 2

Differentiation of equation (3) yields

$$\begin{aligned} \bar{e}_1(\kappa, w, \lambda) &= \frac{w \pi_1(\bar{e}(\kappa, w, \lambda), \lambda)}{1 - \kappa w \pi_{11}(\bar{e}(\kappa, w, \lambda), \lambda)}; \\ \bar{e}_2(\kappa, w, \lambda) &= \frac{\kappa \pi_1(\bar{e}(\kappa, w, \lambda), \lambda)}{1 - \kappa w \pi_{11}(\bar{e}(\kappa, w, \lambda), \lambda)} = \frac{\kappa}{w} \bar{e}_1(\kappa, w, \lambda). \end{aligned}$$

Set $w = R$ in equation (22) to obtain $w_1^*(\kappa_h, \kappa_l, \lambda) = -w^*(\kappa_h, \kappa_h, \lambda) / \kappa_h$. Then from equation (6),

$$\begin{aligned} e_1^*(\kappa_h, \kappa_l, \lambda) &= \bar{e}_1(\kappa_h, w^*(\kappa_h, \kappa_l, \lambda), \lambda) + \bar{e}_2(\kappa_h, w^*(\kappa_h, \kappa_l, \lambda), \lambda) w_1^*(\kappa_h, \kappa_l, \lambda) \\ &= \bar{e}_1(\kappa_h, w^*(\kappa_h, \kappa_l, \lambda), \lambda) \left\{ 1 - \frac{\kappa_h}{w^*(\kappa_h, \kappa_l, \lambda)} \cdot \frac{w^*(\kappa_h, \kappa_l, \lambda)}{\kappa_h} \right\} = 0. \end{aligned}$$

Similarly, differentiating equation (5) yields

$$w_2^*(\kappa_h, \kappa_l, \lambda) = R\pi(\bar{e}(\kappa_l, R, \lambda), \lambda) / \{\kappa_h\pi(\bar{e}(\kappa_h, w_2^*(\kappa_h, \kappa_l, \lambda), \lambda), \lambda)\}.$$

Since $e_1^* = 0$ we have $e^*(\kappa_l, \kappa_l, \lambda) = e^*(\kappa_h, \kappa_l, \lambda)$ and hence $w_2^*(\kappa_h, \kappa_l, \lambda) = R/\kappa_h$. Hence

$$\begin{aligned} e_2^*(\kappa_h, \kappa_l, \lambda) &= \bar{e}_2(\kappa_h, w_2^*(\kappa_h, \kappa_l, \lambda), \lambda) w_2^*(\kappa_h, \kappa_l, \lambda) \\ &= \frac{R\pi_1(\bar{e}(\kappa, w, \lambda), \lambda)}{1 - \kappa_h w_2^*(\kappa_h, \kappa_l, \lambda) \pi_{11}(\bar{e}(\kappa, w, \lambda), \lambda)} > 0. \end{aligned}$$

Now differentiate equation (10) to obtain

$$\begin{aligned} \pi_1^*(\kappa_h, \kappa_l, \lambda) &= \pi_1(e^*(\kappa_h, \kappa_l, \lambda), \lambda) e_1^*(\kappa_h, \kappa_l, \lambda) = 0; \\ \pi_2^*(\kappa_h, \kappa_l, \lambda) &= \pi_1(e^*(\kappa_h, \kappa_l, \lambda), \lambda) e_2^*(e^*(\kappa_h, \kappa_l, \lambda), \lambda) \\ &= \frac{R[\pi_1(\bar{e}(\kappa, w, \lambda), \lambda)]^2}{1 - \kappa_h w_2^*(\kappa_h, \kappa_l, \lambda) \pi_{11}(\bar{e}(\kappa, w, \lambda), \lambda)} > 0. \end{aligned}$$

Differentiation of equation (8) and using equations (20) and (21) yields

$$\begin{aligned} I_1^*(\kappa_h, \kappa_l, \lambda) &= \bar{I}_1(\kappa_h, w_2^*(\kappa_h, \kappa_l, \lambda), \lambda) + \bar{I}_2(\kappa_h, w_2^*(\kappa_h, \kappa_l, \lambda), \lambda) w_1^*(\kappa_h, \kappa_l, \lambda) \\ &= w_2^*(\kappa_h, \kappa_l, \lambda) \pi(e^*(\kappa_h, \kappa_l, \lambda), \lambda) + \kappa_h \pi(e^*(\kappa_h, \kappa_l, \lambda), \lambda) (-w_2^*(\kappa_h, \kappa_l, \lambda) / \kappa_h) = 0; \\ I_2^*(\kappa_h, \kappa_l, \lambda) &= \bar{I}_2(\kappa_h, w_2^*(\kappa_h, \kappa_l, \lambda), \lambda) w_2^*(\kappa_h, \kappa_l, \lambda) \\ &= R\pi^*(\kappa_h, \kappa_l, \lambda) > 0. \end{aligned}$$

For the second part of the proposition, it is simplest to express P^* as in equation (12). Differentiate to obtain:

$$\begin{aligned} P_1^*(\kappa_h, \kappa_l, \lambda) &= R\pi^*(\kappa_h, \kappa_l, \lambda) + R\Delta\kappa\pi_1^*(\kappa_h, \kappa_l, \lambda) = R\pi^*(\kappa_h, \kappa_l, \lambda) > 0; \\ P_2^*(\kappa_h, \kappa_l, \lambda) &= R[\Delta\kappa\pi_2^*(\kappa_h, \kappa_l, \lambda) - \pi^*(\kappa_h, \kappa_l, \lambda)] \\ &= R\left[\frac{R\Delta\kappa[\pi_1(e^*(\kappa_h, \kappa_l, \lambda), \lambda)]^2}{1 - \kappa_l R\pi_{11}(e^*(\kappa_h, \kappa_l, \lambda), \lambda)} - \pi^*(\kappa_h, \kappa_l, \lambda)\right] \\ &< 0 \text{ by assumption (7).}^4 \end{aligned}$$

Equation (11), immediately yields

$$\begin{aligned} \mathcal{W}_1^*(\kappa_h, \kappa_l, \lambda) &= I_1^*(\kappa_h, \kappa_l, \lambda) + P_1^*(\kappa_h, \kappa_l, \lambda) > 0; \\ \mathcal{W}_2^*(\kappa_h, \kappa_l, \lambda) &= \frac{\Delta\kappa[R\pi_1(e^*(\kappa_h, \kappa_l, \lambda), \lambda)]^2}{1 - \kappa_l R\pi_{11}(e^*(\kappa_h, \kappa_l, \lambda), \lambda)} > 0. \end{aligned}$$

Proof of Proposition 3

Firstly, note from lemma 1 that $w^*(\kappa_h, \kappa_l, \lambda) = \kappa_l R / \kappa_h$ and hence that $w_3^* = 0$. From equation (8),

$$\begin{aligned} I_3^*(\kappa_h, \kappa_l, \lambda) &= \bar{I}_2(\kappa_h, w^*(\kappa_h, \kappa_l, \lambda), \lambda) w_3^*(\kappa_h, \kappa_l, \lambda) + \bar{I}_3(\kappa_h, w^*(\kappa_h, \kappa_l, \lambda), \lambda) \\ &= \kappa_h w^*(\kappa_h, \kappa_l, \lambda) \pi_2(e^*(\kappa_h, \kappa_l, \lambda), \lambda) > 0. \end{aligned} \quad (23)$$

Now differentiate equation (3) to obtain

$$\bar{e}_3(\kappa, w, \lambda) = \frac{\kappa w \pi_{12}(\bar{e}(\kappa, w, \lambda), \lambda)}{1 - \kappa w \pi_{11}(\bar{e}(\kappa, w, \lambda), \lambda)}. \quad (24)$$

From equation (6) we have

$$\begin{aligned} e_3^*(\kappa_h, \kappa_l, \lambda) &= \bar{e}_2(\kappa_h, w^*(\kappa_h, \kappa_l, \lambda), \lambda) w_3^*(\kappa_h, \kappa_l, \lambda) + \bar{e}_3(\kappa_h, w^*(\kappa_h, \kappa_l, \lambda), \lambda) \\ &= \bar{e}_3(\kappa_h, w^*(\kappa_h, \kappa_l, \lambda), \lambda), \end{aligned} \quad (25)$$

from which part 2 follows immediately.

From equation (12) $P_3^*(\kappa_h, \kappa_l, \lambda) = R \Delta \kappa \pi_3^*(\kappa_h, \kappa_l, \lambda)$, so that $P_3^*(\kappa_h, \kappa_l, \lambda)$ has the same sign as $\pi_3^*(\kappa_h, \kappa_l, \lambda)$. Equation (10) has the following λ derivative:

$$\pi_3^*(\kappa_h, \kappa_l, \lambda) = \pi_1(e^*(\kappa_h, \kappa_l, \lambda), \lambda) e_3^*(\kappa_h, \kappa_l, \lambda) + \pi_2(e^*(\kappa_h, \kappa_l, \lambda), \lambda), \quad (26)$$

from which condition (13) is immediate. Condition (14) is obtained by substituting for $e_3^*(\kappa_h, \kappa_l, \lambda)$ using equations (24) and (25).

The proof of part 4 is entirely analogous to that of part 3.

Proof of Lemma 2

Recall from equation (23) that $I_3^*(\kappa_h, \kappa_l, \lambda) = \kappa_l R \pi_2(e^*(\kappa_h, \kappa_l, \lambda), \lambda)$. Hence

$$I_{33}^*(\kappa_h, \kappa_l, \lambda) = \kappa_l R [\pi_{12}(e^*(\kappa_h, \kappa_l, \lambda), \lambda) e_3^*(\kappa_h, \kappa_l, \lambda) + \pi_{22}(e^*(\kappa_h, \kappa_l, \lambda), \lambda)].$$

Substituting for $e_3^*(\kappa_h, \kappa_l, \lambda)$ using equations (24) and (25) and rearranging, gives us $I_{33}^*(\kappa_h, \kappa_l, \lambda) < 0$ if and only if

$$\kappa_h w^*(\kappa_h, \kappa_l, \lambda) \left[\pi_{11} \pi_{12} - (\pi_{12})^2 \right] - \pi_{22} > 0, \quad (27)$$

where every π derivative is evaluated at $(e^*(\kappa_h, \kappa_l, \lambda), \lambda)$. The square-bracketed term in condition (27) is positive by concavity of π and so the condition is always satisfied.

Equation (12) implies that $P_{33}^*(\kappa_h, \kappa_l, \lambda) = R\Delta\kappa\pi_{33}^*(\kappa_h, \kappa_l, \lambda)$. Differentiation of equation (26) yields

$$\pi_{33}^* = \pi_{11} (e_3^*)^2 + 2\pi_{12}e_3^* + \pi_{22} + \pi_1 e_{33}^*,$$

where as usual every term involving π is evaluated at $(e^*(\kappa_h, \kappa_l, \lambda), \lambda)$ and every term in e^* is evaluated at $(\kappa_h, \kappa_l, \lambda)$. Rearranging this expression yields condition (15).

Proof of Lemma 4

From equation (16), $L_1^e(\kappa_h, \kappa_l) = \frac{I_{31}^*(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l))}{C''(L^e(\kappa_h, \kappa_l)) - I_{33}^*(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l))}$, which is zero since $I_1^* \equiv 0$. Similarly, $L_2^e(\kappa_h, \kappa_l) = \frac{I_{32}^*(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l))}{C''(L^e(\kappa_h, \kappa_l)) - I_{33}^*(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l))}$. $I_{32}^* = R\{\pi_2(e^*, \lambda) + \kappa_l\pi_{12}(e^*, \lambda)e_2^*\}$, where all starred functions are evaluated at $(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l))$. Substituting $e_2^* = e_3^* \frac{\pi_1(e^*, \lambda)}{\pi_{12}(e^*, \lambda)\kappa_l}$ we obtain $I_{32}^* = R\{\pi_2(e^*, \lambda) + \pi_1(e^*, \lambda)e_3^*\}$ and hence that $I_{32}^* > 0$ if and only if condition (13) is satisfied: this is assumed to be the case. Hence $L_2^e(\kappa_h, \kappa_l) > 0$.

Differentiate equation (17) to obtain $L_1^f(\kappa_h, \kappa_l) = \frac{P_{31}^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l))}{C''(L^f(\kappa_h, \kappa_l)) - P_{33}^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l))}$, and observe that $P_{31}^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l)) = R\{\pi_3^* + \Delta\kappa\pi_{31}^*\} = R\pi_3^*$, where we use the fact that $\pi_1^* \equiv 0$. Hence, invoking assumption (15), $L_1^f(\kappa_h, \kappa_l) > 0$.

Finally, $L_1^f(\kappa_h, \kappa_l) = \frac{P_{31}^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l))}{C''(L^f(\kappa_h, \kappa_l)) - P_{33}^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l))}$. Condition (19) follows by differentiating equation (12).

Proof of Proposition 4

Using the fact that $I_1^* \equiv 0$ and the Envelope Theorem, $E_1^e(\kappa_h, \kappa_l) = 0$. Also,

$$E_1^f(\kappa_h, \kappa_l) = I_3^* \left(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l) \right) L_1^f(\kappa_h, \kappa_l) > 0.$$

Since $L^f(\kappa_l, \kappa_l) = 0$ and $L^e(\kappa_h, \kappa_l) = L^e(\kappa_h, \kappa_l)$ if and only if $E^e(\kappa_h, \kappa_l) \geq E^f(\kappa_h, \kappa_l)$ the existence of $\bar{\kappa}_h(\kappa_l)$ follows immediately.

To show that $\bar{\kappa}_h'(\kappa_l) > 0$, it is sufficient to show that $E^e(\kappa_h, \kappa_l) - E^f(\kappa_h, \kappa_l)$ is increasing in κ_l when $\kappa_h = \bar{\kappa}_h(\kappa_l)$. We have $E_2^e(\kappa_h, \kappa_l) = I_2^*(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l))$ and $E_2^f(\kappa_h, \kappa_l) = I_2^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l)) + I_3^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l))L_2^f(\kappa_h, \kappa_l) < I_2^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l))$, since $I_3^* > 0$ and by assumption (19), $L_2^f(\kappa_h, \kappa_l) < 0$. We prove in lemma 4 that $I_{23}^* > 0$ and we know that $L^e(\kappa_h, \kappa_l) > L^f(\kappa_h, \kappa_l)$ at $\bar{\kappa}_h$, so finally $E_2^f(\kappa_h, \kappa_l) < E_2^e(\kappa_h, \kappa_l)$, as required.

For $\kappa_h < \bar{\kappa}_h(\kappa_l)$, $\mathcal{W}(\kappa_h, \kappa_l) = E^e(\kappa_h, \kappa_l) + F^e(\kappa_h, \kappa_l)$: since $E_1^e(\kappa_h, \kappa_l) = 0$ we have

$$\begin{aligned} \mathcal{W}_1(\kappa_h, \kappa_l) &= F_1^e(\kappa_h, \kappa_l) \\ &= P_1^*(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l)) + P_3^*(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l)) L_1^e(\kappa_h, \kappa_l) \\ &= P_1^*(\kappa_h, \kappa_l, L^e(\kappa_h, \kappa_l)) > 0, \text{ by lemma 4.} \end{aligned}$$

For $\kappa_h > \bar{\kappa}_h(\kappa_l)$, $\mathcal{W}(\kappa_h, \kappa_l) = E^f(\kappa_h, \kappa_l) + F^f(\kappa_h, \kappa_l)$. We have shown that $E_1^f(\kappa_h, \kappa_l) > 0$. Using the Envelope Theorem, $F_1^f(\kappa_h, \kappa_l) = P_1^*(\kappa_h, \kappa_l, L^f(\kappa_h, \kappa_l)) > 0$ also.

Finally, note that $E^e(\kappa_h, \kappa_l) = E^f(\kappa_h, \kappa_l)$ when $\kappa_h = \bar{\kappa}_h(\kappa_l)$ and hence that $L^e(\kappa_h, \kappa_l) > L^f(\kappa_h, \kappa_l)$. It follows that $F^f(\kappa_h, \kappa_l) < F^e(\kappa_h, \kappa_l)$ and hence that \mathcal{W} drops by $F^f(\kappa_h, \kappa_l) - F^e(\kappa_h, \kappa_l)$ at $\bar{\kappa}_h(\kappa_l)$.