Buy-Side and Sell-Side: The Industrial Organization of Information Production in the Securities Industry

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Abstract

This paper studies how information production is allocated between buy-side and sell-side when identical information-producing agents can choose to be either sell-side analysts or buy-side fund managers. For analysts to be in equilibrium, an investment banking subsidy to analysts is necessary, because without subsidy competition among both analysts and fund managers makes an analyst’s profit lower than a fund manager’s. Tying investment banking to sell-side research enables such a subsidy. Because analysts improve social welfare by enhancing the information efficiency of the financial markets, a total separation of investment banking from sell-side research to solve the conflicts of interest may not be a good idea. This paper also explains the existence of independent research and the fee structure in the credit rating industry.

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1 Introduction

A major function of the securities industry is investment research. Analysts produce information to identify securities whose prices do not accurately reflect their future values. The many organizations providing investment research fall mainly into two categories: buy-side and sell-side firms.\footnote{There are also \textit{independent} firms, whose main business is selling research, such as Sanford Bernstein and Value Line. According to Cheng, Liu and Qian (2003), 71\% of research is produced by buy-side firms, 24\% by sell-side firms, and 5\% done by independent firms.} In general, buy-side firms are assets management firms, and sell-side firms are brokerage firms. Research generated by a buy-side firm (buy-side research) is used exclusively for the firm’s trading; research generated by a sell-side firm (sell-side research) is, however, disseminated among the firm’s clients. Buy-side firms profit from trading on both buy-side and sell-side research; sell-side firms profit from selling their research, bundled in most cases with brokerage and investment banking services.

Although information production is important to the securities industry, there have been few attempts made to understand the industrial organization of it. Some important questions are left unanswered as a result. For example, why is information produced by both buy-side and sell-side firms? How is information production allocated between the sell-side and the buy-side? How is buy-side and sell-side research used by the buy-side firms? What value does sell-side research add to the buy-side firms? Why is sell-side research linked to investment banking? What will happen if the research function are completely separated from investment banking? This paper propose to answer these questions in an effort to understand the industrial organization of information production in the securities industry.

These questions also have important public policy implications. In 2003, New York Attorney General Elliot Spitzer, and later the Securities and Exchange Commission, accused investment bankers of pandering to corporate issuers by pressuring sell-side analysts to recommend the stocks of various Internet and telecommunications companies. Despite so-called Chinese wall restrictions at investment banks, at the center of these charges is a conflict of interest between a firm’s research...
division and investment banking division. More specifically, it is a conflict of interest between the obligation of sell-side research to provide investors with fair, unbiased, and sometimes critical research and the expectation that analysts will help investment bankers win new business or at least not impede their efforts. One tempting solution is to totally separate sell-side research from investment banking. Such a separation, not very difficult to implement, will eliminate the source of the conflict of interests. But is this a good idea? My analysis suggests that it may not be, however appealing the motivation. Separating sell-side research from investment banking will cause the end of sell-side research, which may diminish social welfare.

In my model, there are a fixed number of identical risk-neutral information-producing agents who can gather information about a traded risky asset. Each agent can choose whether to be an analyst (i.e., on the sell-side) or a fund manager (i.e., on the buy-side). Fund managers produce their own information, and may buy information from the analysts. Using that information, fund managers then trade in the financial market. Analysts profit from selling information and also from other exogenous businesses, called “investment banking”; fund managers profit from trading on both buy-side and sell-side information. I assume that fund managers cannot sell any information, either their own or information they buy from analysts, and that analysts cannot trade in the financial market. In my model, the quality of information is verifiable, so there is no agency problem in selling.

My main result is that, without profit from other business, an analyst’s profit from selling information is so low compared to a fund manager’s that no information-producing agent is willing to be an analyst in equilibrium. There are two factors that reduce the profit from selling information. One is competition among fund managers: Because fund managers compete to benefit from an analyst’s information in the financial market, they compete away the total expected trading profit generated by the analyst’s information. The other is competition among analysts: Because analysts compete to sell information to fund managers, analysts capture only a fraction of the overall marginal

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2 Under the Security Act of 1933 and the Security Exchange Act of 1934, the SEC endorses the separation of the research departments from other departments in a brokerage firm.
trading profit their information generates, leaving the rest to the fund managers. Without subsidy from other business, these two effects make an analyst’s profit so unfavorable compared to a fund manager’s that an information-producing agent is always better off choosing to be a fund manager. Therefore, for analysts to be in equilibrium, some subsidy to the analysts is necessary. Tying investment banking to sell-side research enables such a subsidy.

Furthermore, I show that having more analysts may enhance social welfare, since more analysts may increase the information content of the asset prices, enabling firms to invest more efficiently. As a result, totally separating investment banking from sell-side research may diminish social welfare because that action may mean the end of sell-side research. I also show that a sell-side analyst exerts more price impact and generates more trading volume than a fund manager. In an extension of the model, I demonstrate that independent research firms, which operate to sell information alone, can survive by providing basic and factual information (as opposed to human analyses provided by sell-side firms). Applying my analysis to the credit rating industry, I provide an explanation of why credit rating agencies charge bond issuers, rather than investors, for their rating services.

Much of the related literature studies a variety of issues in information selling. Admati and Pfleiderer (1986), (1988b), and (1990) show that for a monopolistic information owner, selling information through a fund (i.e., selling information indirectly) is more profitable than selling it to investors who then trade in the financial markets (i.e., selling information directly). Brennan and Chordia (1991) in a study of information selling by brokers show that charging investors brokerage commissions is a way for investors to share risk with risk-neutral brokers. Fishman and Hagerty (1995) demonstrate that selling information directly can arise when there are multiple competing informed traders. By selling his information to others, one informed trader can commit to trade more aggressively on the information, thus making a higher profit. Vishny (1985) studies a brokerage firm’s incentive to sell information in order to increase the liquidity of the market. If liquidity traders trade more as the market becomes more liquid, and if the brokerage firm can make enough on trading commissions, it is optimal for the brokerage firm to sell information, even though doing so will devalue the information.
Another strand of literature focuses on the reliability problem in selling information. Bhattacharya and Pfleiferer (1985) study the problem of mutual funds and other institutional investors in ensuring that people with information gathering ability actually manage their portfolios. Allen (1990) examines a setting in which risk aversion is not observable and the financial market is competitive. Both information buyers and seller are better off participating in the market for financial information, but the seller cannot capture the entire value of the information because of the reliability problem. Heinkel and Stoughton (1994) consider a fund manager’s incentive when the manager cares about his reputation but doesn’t want to exert effort to generate information about trading opportunities. The optimal contract ensures that the efficient level of managerial effort is exerted.

In the rest of the paper, section 2 introduces the model. Section 3 characterizes the market for information and information-producing agents’ specialization decisions. Section 4 studies the relation between sell-side research and information efficiency of the financial market. Section 5 extends the model to explain why independent research exists. Section 6 discusses implications of the model, with emphasis on credit rating industry. Section 7 concludes. Detailed proofs are provided in the appendix.

2 The Model

In a four-dates-three-periods economy, there is one risky asset. At the end of the third period, date 3, the payoff of the asset, \( \delta + V \) is realized. \( V \) is a known constant, and \( \delta \) is random. The prior distribution of \( \delta \) at date 0 is \( N(0, \sigma_\delta^2) \). As a convention, I call \( \frac{1}{\sigma_\delta^2} \) precision and denote it \( v_\delta \). Without loss of generality, I assume \( v_\delta = 1 \).

2.1 Agents and Information

There are three kinds of agents in the economy: information-producing agents, market makers, and liquidity traders. All of them are risk-neutral.
There are $N$ information-producing agents in the economy. $N$ is exogenous for simplicity. Before date 2 but after date 1, each information-producing agent receives one signal about the asset value. The signal of agent $i$, $i = 1, 2, ..., N$, takes the form:

$$s_i = \delta + \varepsilon_i$$  \hspace{1cm} (1)

where $\varepsilon_i \sim N(0, \sigma_i^2)$. For ease of notation, denote $v_i \equiv \frac{1}{\sigma_i^2}$. $\delta$ and $\varepsilon_i$ are independent for any $i$. $\varepsilon_i$ is also independent across information producing-agents. This assumption captures the feature that each agent has a unique perspective about the asset value. I further assume that all signals have the same quality; that is, $v_i = v_j = v$ for any $i$ and $j$. For simplicity I assume that each agent’s cost of receiving the signal is zero.\footnote{If there is a positive cost, each agent would decide whether it is worthwhile to become informed. Apart from this, the analysis would be similar.}

The market makers set the trading price in the financial market at date 2. I assume that the liquidity traders trade in the financial market at date 2 for liquidity reasons. Their demand of the asset is $z$, which is normally distributed with mean zero and variance $\sigma_z^2$.

### 2.2 Sequence of Events

At date 0, each information-producing agent specializes, i.e., chooses whether to be an analyst or a fund manager.\footnote{I assume there is coordination between agents. To be more specific, there is some coordination device that tells each information-producing agent whether he should be an analyst or a fund manager. In equilibrium, each agent should find it optimal to follow the suggestion of this coordination device.} An analyst cannot trade in the financial market, but can sell information to fund managers and engage in other profitable activities, which are called “investment banking”.\footnote{Although many sell-side firms have asset management divisions, presumably there are Chinese wall restrictions that separate sell-side research and buy-side research. Unlike the relation between sell-side research and investment banking, the relation between buy-side and sell-side research doesn’t seem to be significant.}\footnote{In the real world, sell-side analysts’ ability to generate investment banking business seems to derive from their influence over buy-side investors (institutional investors, the fund managers in our model). It is highly unlikely that a large company would steer any of its investment banking business, whether raising capital or handling an M&A assignment, to a Wall Street firm that didn’t cover its stock.} Profit from investment banking is $\pi_I$, which is exogenous in the model. A fund manager, on the other
hand, cannot sell any information, either his own or any bought from analysts, but can trade in the financial market on his own information and information he chooses to buy from analysts.\footnote{The assumption that fund managers cannot resell analysts’ information is crucial for the existence of a market for information. Without the ability to enforce their intellectual property, analysts cannot make any profit selling information. This assumption seems to be consistent with reality, because there are firms that are successful in selling information, such as Sanford Bernstein. We don’t see many funds selling information, possibly because it may create opportunities for funds to profit from selling information at the cost of their investors.}  After the information-producing agents specialize, there are \( m \) analysts and \( n \equiv N - m \) fund managers (\( m \) and \( n \) are determined endogenously). I assume the information producing agents observe each other’s specialization decisions.

At date 1, before they receive signals about the asset value, analysts try to sell their information to fund managers in the market for information. Analyst \( j \) (\( j = 1, 2, \ldots, m \)) makes take-it-or-leave-it offers to a subset of the fund managers of his choice, \( F_j \), for his information at prices \( p(F_j) \). \( p(F_j) \) is a vector whose elements are \( p_{ij} \), \( i \in F_j \). \( p_{ij} \) is the offer price analyst \( j \) demands from fund manager \( i \) for the information. After analysts make their offers, fund manager \( i \) (\( i = 1, 2, \ldots, n \)) receives offers from a subset of analysts, \( S_i \), with offering prices \( p(S_i) \). \( P(S_i) \) is a vector whose elements are \( p_{ij} \), \( j \in S_i \). Fund manager \( i \) then decides whether to accept any of the \( S_i \) analysts’ offers. If he decides to accept the offer from analyst \( j \), an information sale goes through; i.e., the fund manager will pay analyst \( j \) \( p_{ij} \), and in exchange analyst \( j \) will report his signal to fund manager \( i \) once the signal is received. If fund manager \( i \) decides not to buy from analyst \( j \), the information sale fails, i.e., fund manager \( i \) will pay nothing to analyst \( j \), and analyst \( j \) will not report his signal to fund manager \( i \). The set of analysts fund manager \( i \) choose to buy from is \( A_i \), and \( A_i \) is a subset of \( S_i \). Notice that in this model there is no agency problem (such as conflict of interest) in information selling, and analysts have all the bargaining power.\footnote{Including conflicts of interest in the model will only strengthen my main results. The incentive for analysts to mislead investors would diminish the quality and thus the value of the analysts’ information in equilibrium (see Crawford and Sobel (1982)), which makes investors less willing to pay for the information, hence render the analysts’ profit from selling information even less.} \footnote{Giving fund managers more bargaining power will only strengthen my results, because this reduces an analyst’s profit but increases a fund manager’s.}

After date 1, each information-producing agent receives a signal about the value of the asset. The analysts then report their signals to the fund managers who have paid them. At date 2,
Each of the $N$ agents decides to be a fund manager or an analyst. After their decisions, $n$ and $m$ are determined.

All signals realized: analysts report their signals to appropriate fund managers.

Asset value is realized.

Figure 1: Sequence of Events

the fund managers trade in the financial market on their information. As mentioned before, the analysts do not participate in the financial market. The trading mechanism is similar to that in Kyle (1985). The fund managers do not observe current price or current quantities traded by other fund managers or by liquidity traders. Market makers do not receive any private information or observe individual quantities traded by the fund managers and the liquidity traders, but they do observe the total order flow from all market participants. Each trader submits a market order, and the market makers clear the market by supplying liquidity at a price conditioning on the total order flow. I further assume that the market makers don’t observe the composition of analysts in the economy, i.e., they don’t know $m$, but they infer $m$ correctly in equilibrium.

Finally, at date 3, the security value is realized and distributed. The sequence of events is summarized in Figure 1.
2.3 Definition of Equilibrium

The equilibrium concept I use here is *Perfect Bayesian Equilibrium*. Formally, an equilibrium consists of components as follows:

(i) Each agent’s choice of whether to be a sell-side analyst or a fund manager.

(ii) The number of analysts, $m$.

(iii) For analyst $j$, the set of fund managers to offer to, $F_j$, and the offer prices $p(F_i)$.

(iv) For fund manager $i$, the set of analysts to buy information from, $A_i$.

(v) The market order submitted by each fund manager to the market makers.

(vi) The market makers’ pricing conditioning on total order flow.

(vii) The beliefs of all agents in the economy.

In an equilibrium, a) conditioning on other people’s equilibrium strategy, each analyst (fund manager) should find it optimal to be an analyst (fund manager); b) conditioning on his information, each fund manager submits an order to maximize his expected trading profit; c) sell-side analyst $j$ chooses $F_j$ and prices $p(F_j)$ to maximize his profit, given his belief; d) fund manager $i$ chooses to buy information from analysts $A_i$ to maximize the expected payoff; e) each market maker sets the trading price conditioning on total order flow to maximize his expected payoff; and f) all beliefs are consistent with the equilibrium strategies of all the agents in the model.

3 Equilibrium Analysis

For simplicity, in the following analysis I focus on symmetric pure strategy equilibria in which all agents of the same type, analysts, fund managers, and market makers, adopt the same strategy. I also analyze trading in the financial market only in the normal-linear framework for the same reason. I solve the equilibrium by backward induction. I first analyze the trading game, i.e., the fund managers’ trading strategy and the market makers’ pricing rule at date 2. Then I analyze
the date 1 trading in the market for information. Finally, I characterize the date 0 specialization decisions of the information-producing agents and the equilibrium number of analysts.

In the financial market, fund managers trade on both their own information and information they bought from analysts. Each fund manager’s expected trading profit thus depends on his own information and the information he bought. Knowing the impact of his information on fund managers’ profits, an analyst at date 1 strategically makes offers to fund managers to extract the maximum payment for his information. At date 0, each information-producing agent makes a specialization decision by weighing the expected profit to be gained as an analyst or as a fund manager. Because an analyst’s profit and a fund manager’s depend on the number of analysts in the economy, the equilibrium number of analysts has to be such that an analyst’s profit is close enough to a fund manager’s so that no analyst want to deviate to be a fund manager (and the converse). Thus the equilibrium analyst composition is determined.

3.1 Trading in Financial Market

At date 2, the participants in the financial market are: $n$ fund managers, liquidity traders, and market makers. I first take as a given that each fund manager buys information from all analysts at date 1 (I show later that this is the equilibrium outcome). Therefore, when he trades in the financial market, fund manager $i$’s information set $F_i$ is: $F_i = \{s_i, s_1, ..., s_m\}$. In words, each fund manager has his own signal, $s_i$, and $m$ signals he bought from analysts. Fund manager $i$ ($i = 1, 2, ..., n$) submits a market order, $x_i$, base on this information to the market makers to maximize the expected profit. The market makers observe only the total order flow, $y = \sum_{i=1}^{n} x_i + z$, and with information extracted from that, establish the market clearing prices, $P_2(y)$. Thus, fund manager $i$’s problem is

$$Max_{x_i} E[x_i(V + \delta - P_2(\sum_{k=1}^{n} x_k + z))|F_i].$$

(2)
Each fund manager takes the market makers’ pricing rule $P_2(\cdot)$ and other fund managers’ trading strategies as given, but exploits his information advantage by accounting for the impact of his trading decision on the price eventually set by the market makers at date 2.

I do not model the market makers’ strategic behavior directly; rather I only restrict it to be linear:

$$P_2(y) = \eta y + V,$$

where $\eta > 0$. By this way, I would like to show that my results do not depend on the specific market clearing mechanism, as long as the market clearing price is linear in total order flow. This formulation includes the well-known case when the market makers are perfectly competitive:$^{10}$

$$P_2(y) = E[\delta|y] + V.$$ (4)

For the rest of the paper, I keep it as a special example.

I propose that fund manager $i$’s trading strategy is of the form:

$$x_i = \beta s_i + a(\sum_{j=1}^{m} s_j),$$ (5)

where $\beta$ and $a$ can be thought of as measures of the aggressiveness with which a fund manager trades on his own information and on sell-side information, respectively.

Without loss of generality, I can rewrite fund manager $i$’ strategy as

$$x_i = \beta s_i + \alpha s_p,$$ (6)

where $s_p = \frac{1}{m} \sum_{j=1}^{m} s_j$ and $\alpha = am$. Note that $s_p$ is the sufficient statistics for $\delta$, given all sell-side signals. I verify in the appendix that above strategy is indeed an equilibrium. It is characterizes by Proposition 1:

$^{10}$It also includes the case where there is no market maker but uninformed risk-averse investors as in Leland (1992).
Proposition 1 (i) There always exists a linear equilibrium for date 2 trading, in which a fund manager’s trading strategy is given by (5), where

\[ a = \frac{2v}{\eta(n+1)(2(1+mv)+(n+1)v)} \]  \hspace{1cm} (7)

\[ \beta = \frac{v}{\eta(2+mv)+(n+1)v} \]  \hspace{1cm} (8)

and the market makers’ pricing rule is given by (3).

(ii) In equilibrium, the expected trading profit of a fund manager is

\[ \pi = \frac{1}{4\eta} \left[ \frac{4v(1+4m+2n+n^2+(1+2m+n)^2v)}{(1+n)^2(2+(1+2m+n)v)^2} \right]. \]  \hspace{1cm} (9)

(iii) The equilibrium asset price is

\[ P_2 = V + \frac{2nv}{(n+1)(2+mv)+(n+1)v} \sum_{j=1}^{m} s_j + \frac{v}{(2+mv)+(n+1)v} \sum_{i=1}^{n} s_i + \eta z. \]  \hspace{1cm} (10)

(iv) If the market makers are perfectly competitive, i.e., (4) holds, then:

\[ a = \frac{2\sigma^2 v}{\sqrt{nD}} \]  \hspace{1cm} (11)

\[ \beta = \frac{\sigma^2 (n+1) v}{\sqrt{nD}} \]  \hspace{1cm} (12)

\[ \eta = \frac{\sqrt{nD}}{\sigma^2 (n+1)(2+mv)+(n+1)v}, \]  \hspace{1cm} (13)

where \( D \equiv 4mv + 4(mv)^2 + 4nv^2 + (n+1)^2(v+v^2). \)

Proposition 1 indicates that fund managers trade on buy-side and sell-side information differently, since \( a \) and \( \beta \) are different. As a result, the two kinds of information have different impacts on the market price. Corollary 2 explores the differences:
Corollary 2 (i) \( \frac{\alpha}{\beta} = \frac{2n}{n+1} \geq 1 \), that is, in aggregate, fund managers trade more aggressively on sell-side information than on buy-side information.

(ii) \( \frac{\alpha}{\beta} = \frac{2n}{n+1} \leq 1 \), that is, a fund manager trade less aggressively on sell-side information than on his own information.

(iii) Ceteris paribus, the price change induced by sell-side information is greater than that induced by buy-side information.

(iv) Ceteris paribus, sell-side information generates more trading volume than buy-side information.

To understand these results, it is important to appreciate the difference between buy-side and sell-side information. Although the two kinds of information are of the same quality, sell-side information is widely known, but buy-side information is known only to the fund managers who produce it. The difference has significant impacts on the trading behavior of fund managers. First, in aggregate, fund managers trade more aggressively on an analyst’s signal than on a fund manager’s. Because all \( n \) fund managers know an analyst’s signal but only one fund manager knows a fund manager’s, there is more competition among fund managers to profit from an analyst’s signal than a fund manager’s. To see why more competition makes fund managers trade more aggressively, let’s compare the effect on the fund managers’ trading of one unit increase in an analyst’s signal with that of one unit increase in a fund manager’s signal. Everything else equal, if there is a one unit increase in an analyst’s signal, a fund manager faces a trade-off: increasing trading by one unit will increase his trading profit, if the increase in trading will not change the price; but the fund manager’s trading will increase the price by \( \eta \) units (in expectation), which will reduce the fund manager’s trading profit. There are, however, \( n \) fund managers trading on the analyst’s signal, thus increasing \( \eta \) units in price will reduce not one but \( n \) fund managers’ trading profit. But because a fund manager cares only about his own benefit, he doesn’t consider the other \( n−1 \) fund managers’ profit loss caused by his increased trading when he trades off the cost and benefit of his trading. In other words, increasing price is an externality caused by one fund manager on the others. It is,
however, a different story when there is a one unit increase in a fund manager’s signal. Because
the fund manager is the only one who is going to trade on the signal, he bears all the costs of the
increase in price caused by his trading, so he considers all the costs and benefits of his trading in
his decision.\textsuperscript{11} Because competition in the former case makes a fund manager puts less weight on
the total costs, but the same weight on the total benefits of his actions as in the latter case, in
aggregate, the fund managers trade more aggressively in the former case than in the latter.

Second, a fund manager trades less aggressively on the sell-side information than on his own
information. Because an analyst’s signal is known to all fund managers, a fund manager takes
other fund managers’ use of this signal into consideration when he makes a trading decision. For
example, if there is a unit increase in an analyst’s signal, as a fund manager is considering increasing
his order, he knows exactly that other fund managers will do the same. Thus the fund manager
knows that the analyst’ signal is going to be incorporated in the eventual trading price even in
the absence of his own trading. On the other hand, when there is a one unit increase in a fund
manager’s signal, he knows other fund managers have no access to it. Therefore, that signal is
going to be incorporated into the price less than the analyst’s signal in the previous case if his order
increase is by the same amount in both cases. Because the fund manager can hide his information
better, he increases his order in a more aggressive response to his own signal than to the analyst’s.

It is worth noticing that $\frac{\alpha}{\beta}$ is decreasing in $n$, and $\frac{\mu n}{\beta}$ is increasing in $n$. Because competition
among fund managers causes them to trade on buy-side and on sell-side information differently, it
should not be surprising that such differences are more pronounced, as there are more fund managers
in the economy. Because the market makers’ pricing rule is linear in total order flow, greater change
in order flow implies greater change in price. Therefore, sell-side research is more influential in
the sense that it can affect the price more. It is often said of sell-side research that it generates

\textsuperscript{11}To be more precise, even in this case there is an externality. To see this, suppose a fund manager increases
his order in response to his own positive signal, the resulting increasing in price will also hurt other fund managers’
trading profit in expectation. The reason is the following: When the fund manager increases his trading, the resulting
increased price reveals other people’s signals too because on average, the other signals are also positive. But the
extent of the externality here is less, as it is caused by the correlation among signals, unlike the externality in the
competition to trade on an analyst’s signal, which is caused by the fact that the fund managers know the analyst’s
signal perfectly.
trade for the brokerage business. This seems to be consistent with my model, because according to Corollary 2, sell-side information generates more trading volume than buy-side information.

### 3.2 Market for Information

Here I study the date 1 market for financial information, more specifically, the analysts’ choices of fund managers to offer to at what prices, and the fund managers’ choice of analysts to buy from. The sequence of events at date 1 is as follows. Each analyst makes offers to sell his information to the fund managers of his choice. Conditioning on the offers a fund manager receives, he decides whether to accept each offer or not. I first characterize analysts’ choice of fund managers to offer, and then fund managers’ demand for analysts’ information. Finally I characterize analysts’ offering prices.

#### 3.2.1 Analysts’ Choice of Fund Managers

One important feature of selling information is that the marginal cost is zero, i.e., the cost of an analyst reporting to one more fund manager is zero. For example, a common way of selling information is through the Internet. To sell to one more fund manager, the only thing an analyst needs to do is to provide the fund manager with access to the website, i.e., set up an user’s account, and give the fund manager a user name and a password. Such activities’ can be carried out at negligible marginal cost. The zero-marginal-cost feature determines that, in equilibrium, an analyst sells his information to all fund managers if he cannot commit to sell to a strict subset of them.

To understand the idea, let’s be more specific about the information structure right after a fund manager receives offers from the analysts. I assume that after a fund manager receives these offers, he knows only whether or not he received an offer from each analyst and if he did, what the offer is. But, the fund manager doesn’t know whether or not an analyst has made offers to other fund managers or what the offers are if they were made. The fund manager then decides whether to
accept each offer or not based on his belief about how many other fund managers will buy the information and the offer price. According to this information structure, I have Lemma 3:

Lemma 3 In a linear and symmetric equilibrium, an analyst successfully sells his information to all fund managers.

The rationale behind Lemma 3 is as follows. Suppose analyst \( q \) successfully sells to only a strict subset of the fund managers, \( F^a_q \), in equilibrium. But analyst \( q \) can do better by deviating to sell his information to one more fund manager, say, \( i \), whom he doesn’t sell to successfully in equilibrium, and keeping the offers to fund managers in \( F^a_q \) the same. By deviating, analyst \( q \) would get the equilibrium payment from fund managers in \( F^a_q \) because these fund managers don’t know that analyst \( q \) has deviated. On top of that, analyst \( q \) can get payment from fund manager \( i \). Therefore, analyst \( q \) is better off by deviating.\(^{12}\) Thus I can conclude that in equilibrium, analysts sell to all fund managers. The fundamental reasons for this result are: (i) analysts cannot commit to sell to only a subset of the fund managers; and (ii) the marginal cost of reporting to one more fund manager is zero.

3.2.2 Fund Managers’ Demand for Sell-Side Information

From the set, \( S^i \), of analysts who made offers to him, how should fund manager \( i \) choose a subset, \( A^i \), of analysts to buy information from?

After fund manager \( i \) chooses \( A^i \), his trading strategy is given by solving the problem:

\[
\max_{x_i} E[x_i(V + \delta - (V + \eta y))|F_i]
\]  

(14)

where \( F_i = \{s_i, s_j, \forall j \in A^i\} \). The information fund manager \( i \) has now includes his own information, \( s_i \), and the analysts’ information it bought, \( s_j, j \in A^i \). Fund manager \( i \) solves problem (14) with the

\(^{12}\)As shown in the appendix, analyst \( q \) is strictly better off by deviating because fund manager \( i \) is willing to pay a positive price for the information; i.e., fund manager \( i \) can make more trading profit with analyst \( q \)’s information than without.
belief that everybody else will play the equilibrium strategy; that is, every other fund managers will buy from all the analysts and trade according to strategy as specified in Proposition 1. Proposition 4 summarizes the solution to the problem:

**Proposition 4** (i) Conditioning on his own information and other fund managers’ equilibrium strategies, fund manager i’s trading strategy is given by

\[
x_i^*(A^i) = \frac{1}{2\eta}[(1 - \eta\beta(n - 1) - \eta\alpha(n - 1)\frac{m - l}{m}) v_i s_l + v s_i - \eta\alpha(n - 1)\frac{l}{m} s_l]
\]

where \(l \equiv l(A^i)\) is the number of analysts in \(A^i\), \(v_l \equiv lv\), and \(s_l \equiv \frac{1}{l} \sum_{j \in A_i} s_j\).

(ii) Fund manager i’s expected trading profit is

\[
\pi_i(A^i) = \frac{1}{4\eta}[(1 - \eta\beta(n - 1) - \eta\alpha(n - 1)\frac{m - l}{m})^2 \frac{v + v_l}{1 + v + v_l} + (\alpha\eta(n - 1)\frac{l}{m})^2 \frac{1 + v_l}{v_l} - 2(1 - \eta\beta(n - 1) - \eta\alpha(n - 1)\frac{m - l}{m})(\alpha\eta(n - 1)\frac{l}{m})]
\]

where \(\alpha\) and \(\beta\) are given by Proposition 1.

Because the equilibrium is symmetric, it is not surprising that fund manager i’s strategy and expected trading profit are functions of only the number of analysts he bought from, not the choice of particular analysts. I can therefore write \(\pi_i(A^i) \equiv \pi_i(l(A^i))\). \(\pi_i(l)\) is the expected trading profit fund manager i can get from buying information from \(l\) analysts, given all other fund managers playing their equilibrium strategies. Notice that \(\pi_i(m) = \pi\), the equilibrium payoff to fund manager i as specified in Proposition 1.

To understand fund manager i’s choice of the number of analysts to buy from, I need to understand the features of \(\pi_i(l)\).

**Corollary 5** Fund manager i’s expected trading profit \(\pi_i(l)\) is strictly increasing and concave in \(l\).

It is not surprising that fund manager i’s profit is increasing in \(l\), since more analysts’ information reduces his information disadvantage relative to other fund managers, who have all the analysts’
information. But the marginal benefit of analysts’ information is decreasing, since as fund manager $i$ gets more analysts’ information, he can make a more precise assessment about the asset’s value and other fund managers’ order flows. The more precise fund manager $i$’s assessment is, the less it is going to be changed by one more analyst’s information. Thus the marginal value of one more analyst’s information is less.

In choosing the optimal set of analysts to buy from $S^i$, fund manager $i$ trades off between the benefit of analysts’ information, given by $\pi_i(l)$, and the cost of it, given by $p(S^i)$. Formally, fund manager $i$’s problem is

$$
\pi_b(S^i) \equiv \max_{A_i \subseteq S_i} \pi_i(l(A^i)) - \sum_{j \in A_i} p^j_i.
$$

(17)

Remember $p^j_i$ is analyst $j$’s offer price to fund manager $i$. I use two steps to solve (17). The first is to determine the optimal $A^i$ for fixed $l$. Because fund manager $i$’s profit depends on only the number of analysts he buys from, not the particular choice of analysts, the optimal $A^i$ satisfies

$$
p^j_i \leq p^k_i, \ \forall \ j \in A^i, \ k \in S^i \setminus A^i.
$$

(18)

In words, if fund manager $i$ decides to buy from $l$ analysts, he chooses to buy from the $l$ cheapest analysts. The second step is to determine the optimal $l$. The monotonicity and concavity of $\pi_i(l)$ make the analysis simple. On the one hand, the marginal benefit of analysts’ information is decreasing; on the other, the marginal cost of the information is increasing because as $l$ increases, more expensive analysts will be included in $A^i$. Thus, the first order condition is necessary and sufficient for the unique optimum for problem (17). The first order condition is

$$
\pi_i(l^* + 1) - \pi_i(l^*) \leq p^*_{l^*+1} \text{ if } l^* < l(S^i),
$$

(19)
where \( p_{l^*} \) (\( p_{l^*+1} \)) denote the \( l^* \)th (\( l^* + 1 \)th) lowest price in \( p(S_i) \). In words, at the optimum, the loss of benefit from excluding one analyst from the \( l^* \) analysts, \( \pi_i(l^*) - \pi_i(l^* - 1) \), is no less than the savings from this action, which is at most \( p_{l^*} \); the additional benefit of buying from one more analyst, \( \pi_i(l^* + 1) - \pi_i(l^*) \), is no greater than the cost of the analyst’s information, which is no less than \( p_{l^*+1} \). Proposition 6 summarizes fund manager \( i \)'s choice of analysts to buy information from:

**Proposition 6** Given the prices of the analysts’ information, \( p(S_i) \), fund manager \( i \) chooses to buy from the \( l^* \) cheapest analysts, and \( l^* \) is determined by (19).

### 3.2.3 Price of Analysts’ Information

There are two factors that determine an analyst’s offering price. First, an analyst successfully sells to all fund managers in equilibrium as established by Lemma 3; second, an analyst makes take-it-or-leave-it offers to the fund managers, thus can extract the marginal surplus of his information from all the fund managers. In a symmetric equilibrium, the marginal benefit of an analyst’s information to a fund manager is \( \pi(m) - \pi(m - 1) \) (we can drop the subscript \( i \) because of symmetry). Therefore, I conjecture that an analyst’s offering price is:

\[
p_j^i = p = \pi(m) - \pi(m - 1), \ \forall i, j.
\]  

(20)

If all the analysts use such a strategy, each fund manager will buy information from all analysts by Proposition 6, so an analyst’s profit from selling information is \( \pi_r \equiv np \) in equilibrium. Why can no analyst benefit from deviating from such a strategy? First, an analyst cannot benefit from offering a lower price to any fund manager, since by Proposition 6 the fund manager would accept the offer, leading to less profit for the analyst. Second, an analyst cannot benefit from offering a higher price to any fund manager either, since by Proposition 6 the fund manager would reject the offer, again leading to less profit for the analyst. Therefore, it is optimal for an analyst to offer \( p \) to all fund managers.
As might be clear by now, the out-of-equilibrium belief supporting the equilibrium is that whatever offers a fund manager receives from the analysts, he believes that all other fund managers buy information from all the analysts at price $p$ and trade as specified in Proposition 1. Proposition 7 characterizes the equilibrium in the market for information:

**Proposition 7** In the market for analysts’ information,

(i) an analyst makes offers to all fund managers to sell information, and the offer price is

$$p = \frac{1}{4\eta} \frac{16v(1 + v + mv)}{(1 + n)^2(1 + mv)(2 + (1 + m + n)v)^2};$$

(ii) all the fund managers accept the offer;

(iii) a fund manager’s and an analyst’s equilibrium profits are

$$\pi_b = \frac{1}{4\eta} \frac{4v[(n + 1)^2 + [4m^2 + (1 + n)^2 + m(1 + n(6 + n))v + m(1 + 2m + n)^2v^2]}{(1 + n)^2(1 + mv)(2 + (1 + m + n)v)^2}$$

and

$$\pi_s = \frac{1}{4\eta} \frac{16nv(1 + v + mv)}{(1 + n)^2(1 + mv)(2 + (1 + m + n)v)^2} + \pi_I(m, n),$$

respectively.

Proposition 7 characterizes the equilibrium payoffs of a fund manager and an analyst as functions of $m$ and $n$. Because $n = N - m$, $\pi_b$ and $\pi_s$ are functions of $m$ only.

### 3.3 Specialization Decision

The next question is the equilibrium composition of analysts and fund managers, that is, the number of information-producing agents who choose to be analysts in equilibrium, $m^*$. Because the number of fund managers is $N - m^*$, $m^*$ also determines the equilibrium composition of fund managers.
In equilibrium, each information-producing agent makes his specialization decision to maximize his expected payoff. Formally, \( m^* \) is an equilibrium composition if and only if

\[
\begin{align*}
\pi_b(m^*) & \geq \pi_s(m^* + 1) \text{ if } m^* \leq N - 1, \quad \text{and} \\
\pi_s(m^*) & \geq \pi_b(m^* - 1) \text{ if } m^* > 0.
\end{align*}
\]

(24)

(25)

(24) is the fund manager’s incentive compatibility condition. It says that if a fund manager (to be more precise, an information producing agent who is supposed to be a fund manager) deviates by becoming an analyst unilaterally, his profit, \( \pi_s(m^* + 1) \), is less than what he can get as a fund manager, \( \pi_b(m^*) \).

Similarly, (25) is the analyst’s incentive compatibility condition. It says that if an analyst deviates by becoming a fund manager unilaterally, his profit, \( \pi_b(m^* - 1) \), is less than what he can get as an analyst, \( \pi_s(m^*) \).

What will happen to the equilibrium composition of analysts if \( \pi_I = 0 \)? That is, will there be analysts in equilibrium if their profit comes from selling research only? The answer is no, as given by the next proposition:

**Proposition 8** If there is no investment banking subsidy, i.e., \( \pi_I = 0 \), then:

(i) An analyst always has incentive to deviate to be a fund manager, i.e.,

\[
\pi_r(m) = \pi_s(m) < \pi_b(m - 1) \quad \forall 1 \leq m \leq N.
\]

(26)

(ii) In equilibrium, \( m^* = 0 \).

Proposition 8 says analysts cannot exist in equilibrium without investment banking profit because the profit an analyst makes by selling information is less than what he can get if he becomes a fund manager. There are two factors that reduce the profit from selling information. The first factor is *competition among fund managers*. Because fund managers compete to benefit from an

\[\text{Notice that } \pi_s(N) = 0 \text{ but } \pi_b(N - 1) > 0, \text{ so } m^* \leq N - 1.\]
analyst’s information in the financial market, they compete away the total expected trading profit generated by the analyst’s information. To see this, notice that Corollary 2 shows that at an aggregate level, fund managers trade more aggressively on an analyst’s signal (with intensity $na$) than on a fund manager’s (with intensity $\beta$). As a result, the analyst’s signal is impounded too much into the price, which makes the total profit generated by the signal less than what it could be if fund managers traded on it less aggressively (with intensity $\beta$). The second factor is *competition among analysts*. Because analysts compete to sell information to fund managers, they capture only a fraction of the overall marginal trading profit their information generates, leaving the rest to the fund managers even though the analysts have all the bargaining power. To see this, recall that Corollary 5 shows a fund manager’s marginal benefit from analysts’s information is decreasing. Because in equilibrium an analyst can extract only the last analyst’s marginal benefit, $p$, from a fund manager as Proposition 7 shows, a fund manager’s total payment to all the analysts, $np$, is only a fraction of the total surplus created by the analysts’ information, as shown in Figure 2. The rest of the surplus is captured by the fund manager.

Proposition 8 points out that investment banking profit is crucial for the sell-side analysts to exist. Proposition 9 gives the properties of the investment banking profit $\pi_I$ that can support a positive number of analysts in equilibrium:

**Proposition 9** $m^*, 1 \leq m^* \leq N - 1$, is the equilibrium number of analysts if and only if

\[
\pi_I(m^* + 1) \leq \pi_b(m^*) - \pi_r(m^* + 1) \tag{27}
\]

\[
\pi_I(m^*) \geq \pi_b(m^* - 1) - \pi_r(m^*) \tag{28}
\]

Notice that by Proposition 8, the right-hand sides of both (27) and 28) are positive. Proposition 9 requires that to have a positive number of analysts in equilibrium, the investment banking profit cannot be too low, otherwise an analyst will have an incentive to be a fund manager. A the same

\[14\text{It can be shown that if all the fund managers can lower } a \text{ so that } na \text{ equals the equilibrium } \beta, \text{ i.e., if fund managers can commit to trade on an analyst’s signal with the same intensity as they do on a fund manager’s, they will earn higher trading profits, } ceteris paribus.\]
Surplus captured by the analysts

Surplus captured by a fund manager

Figure 2: Price of Analysts' Information
time, the investment banking profit $\pi_I(m^* + 1)$ cannot be too great, otherwise a fund manager will have incentive to be an analyst.

Proposition 9 states that in equilibrium analysts have to do other business, such as investment banking, in addition to selling information. It thus explains why sell-side research is usually bundled with investment banking in the real world. Totally separating investment banking and sell-side research to eliminate the conflicts of interest may not be a good idea if regulators want to maintain some number of sell-side analysts. The division of the business may make it impossible for sell-side analysts to survive.

The overall equilibrium is summarized as follows. In equilibrium, $m^*$ information-producing agents specialize as analysts and the rest as fund managers at date 0 as described by Proposition 9. In the market for information, each analyst sells his information successfully to all the fund managers at price $p$ as specified by Proposition 7. Each fund manager buys from all the analysts and then trades in the financial market as described by Proposition 1.

In an equilibrium with a positive number of analysts, is an analyst’s profit from selling information higher or lower than a fund manager’s profit? More precisely, is $\pi_b(m^*) \geq \pi_r(m^*)$? By the intuition of Proposition 8, this should be so. Proposition 10 shows it is actually the case:

**Proposition 10** *In a equilibrium with $m^* \geq 1$,

$$\pi_b(m^*) > \pi_r(m^*).$$

(29)

That is, an analyst’s profit from selling information is always less than a fund manager’s profit.

The empirical prediction of Proposition 10 is that a sell-side firm’s profit from selling information should be less than a fund manager’s profit, controlling for research capacity.
3.3.1 Numerical Illustration

I assume that an analyst’s profit from investment banking is proportional to his profit from selling information, that is

\[ \pi_I = \frac{1 - \rho}{\rho} np, \quad 0 < \rho \leq 1, \]

where \( \rho \) is a constant. The higher the \( \rho \), the lower the investment banking profit proportional to an analyst’s total profit. In the extreme case where \( \rho = 1 \), \( \pi_I = 0 \). I further assume that \( \rho = 0.015 \), \( N = 100 \), and \( v = 0.8 \). I can then calculate \( \pi_b \) and \( \pi_s \) as a function of \( m \), as shown in Figure 3.

In Figure 3, \( \pi_b \) and \( \pi_s \) cross at E. That is, a fund manager and an analyst have the same profit at E. An analyst’s profit, \( \pi_s \), decreases near E, while a fund manager’s profit, \( \pi_b \), increases near E as more information-producing agents become analysts. Thus E is a good candidate for an equilibrium. At this point, an analyst has no incentive to switch to be a fund manager because if he does, a fund manager’s profit will decline, in which case the profit will be less than the profit gained as an analyst. Nor does a fund manager have an incentive to switch, if he does, an analyst’s profit will decline too, in which case the profit will be less than the profit gained as a fund manager.
At $E$, $m^E = 31.37$, but the equilibrium number of analysts, $m^*$, has to be an integer. Therefore, I need to verify whether $m = 31$, or $m = 32$, or both satisfy the equilibrium conditions (24) and (25). It turns out that $m^* = 32$ is an equilibrium number of analysts.\footnote{It can be shown that if $E$ is such that $\pi_r$ is decreasing and $\pi_b$ is increasing in $m$ around $E$, between the two integers closest to $m^E$, one and only one is the equilibrium number of analysts.} It is worth noting that $\rho$ is 0.015, which means that in equilibrium the profit from selling information accounts for only 1.5% of an analyst’s total profit. It can be shown that if $\rho \geq 0.031$, the equilibrium number of analysts is zero ((25) is never satisfied if $\rho \geq 0.031$). Therefore, to have a positive number of analysts in equilibrium, the investment banking profit has to be a significant portion of an analyst’s total profit, at least 96.9%. Further, $\frac{\pi_r(32)}{\pi_b(32)} = 1.48\%$. That is, an analyst’s profit from selling information is only 1.48% of a fund manager’s profit in equilibrium.

### 3.3.2 Limit Results

So far, I have shown that competition among analysts in selling information and among fund managers in trading reduce an analyst’s profit from selling information, $\pi_r$, relative to a fund manager’s profit, $\pi_b$. A priori, as the competition among fund managers, among analysts, or both, increases, $\pi_r$ should decrease relative to $\pi_b$. As a result, $\pi_I$ has to become more important compared to $\pi_r$ in order to support an equilibrium with a positive number of analysts.

To study this formally, I first fix the number of analysts, $m$, and let the number of fund managers, $n$, go to infinity. I am interested in how the ratio, $\frac{\pi_r}{\pi_b}$, will behave at the limit. An increasing number of fund managers indicates that competition among analysts is intensifying. Then I fix $n$ and let $m$ go to infinity, and repeat the analysis. Finally, I let both $m$ and $n$ go to infinity, keeping the proportion of the analysts fixed.

**Proposition 11 (i)**

\[
\lim_{n \to \infty} \frac{\pi_r(m, n)}{\pi_b(m, n)} = \lim_{n \to \infty} \frac{\pi_r(m, n)}{\pi_b(m-1, n+1)} = 0, \quad (30)
\]
\[
\lim_{m \to \infty} \frac{\pi_r(m, n)}{\pi_b(m, n)} = \lim_{m \to \infty} \frac{\pi_r(m, n)}{\pi_b(m-1, n+1)} = 0. \quad (31)
\]
Let \( x \in (0, 1) \) be the level of analysts, i.e., \( m = xN \); then:

\[
\lim_{N \to \infty} \frac{\pi_r(xN, (1 - x)N)}{\pi_b(xN, (1 - x)N)} = \lim_{N \to \infty} \frac{\pi_r(xN, (1 - x)N)}{\pi_b(xN - 1, (1 - x)N + 1)} = 0.
\] (32)

(ii) Therefore, in an equilibrium with a positive level of analysts:

\[
\lim_{m \to \infty} \frac{\pi_r(m, n)}{\pi_I(m, n)} = \lim_{n \to \infty} \frac{\pi_r(m, n)}{\pi_I(m, n)} = \lim_{N \to \infty} \frac{\pi_r(xN, (1 - x)N)}{\pi_I(xN, (1 - x)N)} = 0.
\] (33)

(iii) The limit results above hold if the total amount of information in the economy, \( \Gamma \equiv Nv \), is fixed.

Proposition 11 confirms the intuition. As competition among analysts, fund managers, or both, intensifies, an analyst’s profit from selling information becomes negligible compared to a fund manager’s profit. As a result, in an equilibrium with a positive level of analysts, investment banking has to be the dominant profit source of an analyst to prevent an analyst from switching to be a fund manager.

4 Social Welfare and Sell-side Research

I have shown that investment banking profit is essential for analysts to exist in equilibrium. But why should one care about the number of analysts in the economy? What is wrong if analyst cease to exist as a result of totally separating investment banking from research? I argue that increasing the number of analysts may enhance social welfare for two reasons. First, because asset prices affect corporate investment, more informative asset prices improve economic efficiency, thus enhancing social welfare;\(^{16}\) Second, because analysts’ information is incorporated more in asset prices than fund managers’ information, increasing the number of analysts may augment the information content of asset prices.

\(^{16}\)There is a large literature on the positive relationship between asset prices and corporate investment. See Baker, Stein, and Wurgler (2003) for a review of this literature.
4.1 Information Efficiency and Social Welfare

To study the relationship between the information efficiency of asset prices and corporate investment efficiency, I introduce into my model a firm that sells its assets in the financial markets. Assume at date 0 that the firm has one project. The firm has to invest in the project to increase its production capacity. At date 3, the value per unit of the firm’s production capacity is $V + \delta$, which can be thought of as the future price of the firm’s product. The investment cost of setting up $q$ units of production capacity to the firm is $f(q)$, $f'(q) > 0$ and $f''(q) > 0$. The firm needs to invest $f(q)$ at date 2 after it raises capital from the financial market. The firm’s owners sell the firm’s stock at date 2, and use part of the proceeds to finance the project, retaining the rest for their own consumption. For simplicity, I assume the total shares of the firm’s stock amounts to $q$, which implies that the date 3 price of its stock is $V + \delta$. To model the firm’s equity issuance in a parsimonious way, I assume that the number of shares issued by the firm, thus its production capacity, depends on the date 2 market price of its shares. More specifically, the firm sells its stock through limit orders. So market makers can tell the firm’s order from the overall orders, since the firm is the only one that uses limit orders in the market.\footnote{This assumption is consistent with the fact that firms have to disclose how many shares they are selling in the primary market.} I further assume that (4) holds, i.e., the market makers provide liquidity at the expected value of the stock, conditional on the total order flow. Finally, the firm pays each analyst’s investment banking fee $\pi_I$.\footnote{It is better to think of $\pi_I$ as expected investment banking fees.}

According to these assumptions, the firm’s problem is

$$\max_{q} qP_2 - f(q).$$ \hfill (34)

The optimal $q$ is thus uniquely determined by:

$$q^* = f'^{-1}(P_2).$$ \hfill (35)
Because $f(q)$ is convex, (35) implies that $q^*$ is increasing in $P_2$. Thus, my model captures in a simple way the positive relationship among stock prices, corporate investment, and equity issuance as documented by Baker, Stein, and Wurgler (2003). Because the firm’s decision depends only on the stock price, whether the firm has private information about $\delta$ or not makes no difference in the model. As a result, the firm’s selling has no information content. Therefore all previous analysis of trading and information selling remains unchanged.

Social welfare, $W$, is defined as the sum of all agents’ ex ante expected payoff. It is straightforward to show that

$$W = E[q^*P_2 - f(q^*)].$$

(36)

It shouldn’t be surprising that the social welfare equals the firm’s profit. Because all agents are risk-neutral, trading in the financial market cannot enhance social welfare. One dollar in trading profit is one dollar in trading loss by somebody else, so trading in the financial market is welfare-neutral. Therefore, social welfare has to be created within the firm, as stated by (36).

Because the firm’s profit depends on its stock price, how informative the stock price is can have important welfare implications. In the model, information efficiency is defined as the ex post reduction in uncertainty about stock value provided by the market price.\footnote{The reduction is from the uninformed agents’ point of view, as will be clear soon.} From the market makers’ point of view, $\delta$ can be written as

$$\delta = E[\delta|y] + e.$$

(37)

The uncertain value of the stock can thus be decomposed into two parts: the revealed part $E[\delta|y]$, and the concealed part $e$. The revealed part is the market price minus $V$, therefore the revealed part is public information. The concealed part remains unknown to the uninformed agents, such
as the market makers.\(^{20}\) From Proposition 1, we know \(E[\delta|y] = \eta y\). Further, it is easy to verify that \(e\) and \(y\) are independent; therefore,

\[
1 = \text{Var}[\delta] = \text{Var}[\eta y] + \text{Var}[e]. \tag{38}
\]

I denote \(\Sigma_r \equiv \text{Var}[\eta y]\). \(\Sigma_r\) is therefore the reduction in uncertainty about the value of the asset, and \(\text{Var}[e]\) is the remaining uncertainty about \(\delta\). The greater the \(\Sigma_r\), the less uncertain about \(\delta\) conditioning on the market price, the more informationally efficient the stock price. Because \(\Sigma_r = \text{Var}[P_2]\), the reduction in uncertainty equals the volatility of the stock price. The more volatile the price is, the more information it reveals.

Proposition 12 states the relationship between the information efficiency of the stock price and social welfare:

**Proposition 12** *Social welfare increases with the information efficiency of the stock price.*\(^{21}\)

The intuition is straightforward. Because the firm makes its investment decision on the basis of the stock price, a more informative price improves the quality of the firm’s investment decision.

4.2 Sell-Side Research and Information Efficiency

Because analysts’ information is incorporated more in asset prices than fund managers’ information, increasing the number of analysts may enhance the information efficiency of stock price, and thus social welfare.

By Proposition 1 and after some algebra, I get

\[
\Sigma_r(m) = \frac{(m - N)v[m^3v - m(N - 1)^2v + m^2(2 + (N - 3)v) - (1 + N)^2(2 + v + NV)]}{(1 - m + N)^2(2 + (1 + m + N)v)^2} \tag{39}
\]

\(^{20}\)The informed agents still know more about \(\delta\) than the uninformed even after the trading price is realized.

\(^{21}\)This result may not be true in an incomplete market because more information may destroy risk-averse agents’ incentive to share risk optimally. This is the well-known Hirshleifer Effect (see Hershleifer (1971)). It can be shown, however, that if the market incompleteness is caused by asymmetric information or moral hazard, more information enhances social welfare. Contact author for details.
Increasing $m$ has two offsetting effects on $\Sigma_r$. First, as Corollary 2 shows, more analysts improves information efficiency because analysts’ information is impounded more into the stock prices than fund managers’ information. I call this the sell-side effect. Second, more analysts diminishes information efficiency by reducing competition among fund managers. Because the number of total information-producing agents is fixed in the economy, more analysts means fewer fund managers. Fewer fund managers in turn reduces the competition among fund managers, which in turn leads to less informative prices because the fund managers bid less aggressively (in the aggregate). I call this the competition effect. Depending on which effect dominates, increasing the number of analysts may improve or diminish the information efficiency of the price, as shown in Figure 4.

In Figure 4, $x = \frac{m}{N}$, the level of analysts in the economy. We can see from the figure that when there are many analysts, information efficiency actually decreases with the level of analysts. Also,
when \( v \) is not too large, \( \Sigma_r \) is increasing in \( m \) when \( m \) is small; but \( \Sigma_r \) is decreasing in \( m \) for large \( v \). These observations are true in general, as shown by Proposition 13:

**Proposition 13** (i) For all \( N \) and \( m \), there exists a positive \( \overline{v} \), such that if \( v > \overline{v} \), \( \Sigma_r \) decreases as the number of analysts increases.

(ii) For all \( N \) and \( m \) such that \( m < 1 + N - \sqrt{2(1+N)} \), there exists a positive \( \underline{v} \), such that if \( v < \underline{v} \), \( \Sigma_r \) increases as the number of analysts increases.

(iii) If \( m \) is close to \( N \), \( \Sigma_r \) decreases as the number of analysts increases.

A large \( v \) diminishes the sell-side effect. That is, if each fund manager already has very precise information, the analysts’ information does not change a fund manager’s trading behavior that much. Therefore, if \( v \) is large enough, the competition effect dominates, i.e., \( \frac{\partial \Sigma_r}{\partial m} < 0 \). Conversely, if \( v \) is small enough, it amplifies the sell-side effect. Thus for small enough \( v \), \( \frac{\partial \Sigma_r}{\partial m} > 0 \) if \( m \) is not so large as to let the competition effect dominate. When \( m \) is large, the number of fund managers \( n \) is small. In this case an increase in \( m \) reduces \( n \) significantly (proportionally), which leads to a significant reduction in competition among the fund managers. Moreover, less competition among fund managers means that the extent to which analysts’ information being impounded into prices more than fund managers’ own information is less (recall \( \frac{na}{\beta} = \frac{2n}{n+1} \) decreases with \( n \)), which weakens the sell-side effect. Therefore the competition effect dominates when \( m \) is large, i.e., \( \frac{\partial \Sigma_r}{\partial m} < 0 \).

When \( N \) is very large, however, the ambiguity of increasing \( m \) disappears, as the next proposition shows. In Proposition 14, \( N \) goes to infinity while the total information in the economy \( \Gamma = Nv \) and the analyst level \( x \) are fixed.

**Proposition 14** If the level of analysts \( x = \frac{m}{N} \) is fixed and \( x \in [0, 1) \) :

\[
\lim_{N \to \infty} \Sigma_r = \frac{\Gamma(1+x)}{2 + \Gamma + \Gamma x}.
\]  

Further,

\[
\lim_{N \to \infty} \frac{\partial \Sigma_r}{\partial x} = \frac{2\Gamma}{(2 + \Gamma + \Gamma x)^2}
\]  

31
Figure 5: Information Efficiency: The Limit Case

\[ \lim_{N \to \infty} \frac{\partial \Sigma_r}{\partial \Gamma} = \frac{2(1 + x)}{(2 + \Gamma + \Gamma x)^2}. \]  

(42)

As \( N \) goes to infinity, \( n = xN \) goes to infinity also. When \( n \) is very large, the sell-side effect dominates the competition effect, contrary to when \( n \) is small. Therefore \( \lim_{N \to \infty} \frac{\partial \Sigma_r}{\partial x} > 0 \).

In words, increasing the number of analysts improves the information efficiency of the economy. Moreover, Proposition 14 shows \( \lim_{N \to \infty} \frac{\partial \Sigma_r}{\partial \Gamma} > 0 \), which implies that as the total private information in the economy increases, information efficiency is also enhanced, as shown in Figure 5.

My conclusion is that under some conditions, increasing the number of analysts improves social welfare. Therefore, totally separating investment banking from research in a firm may not be the good idea it seems to be, since the information efficiency of the financial market may decrease, which may diminish social welfare.
5 Extension: Independent Research

In the real world, there are independent research firms whose main business is selling research unlike the sell-side research firms. Examples are Sanford Bernstein, Standard & Poor’s, and Bloomberg. Why can independent research firms exist without the benefit of investment banking profit while sell-side research firms cannot? The answer lies in the different kinds of information that sell-side research and independent research provide. Sell-side research firms focus more on providing human analyses and opinions, such as analysts’ recommendations. Independent research firms focus more on providing basic and factual information, such as company news and history. I call the former direct information and the latter indirect information. Information-producing agents need the indirect information that independent research firms provide to generate their own information. For example, a fund manager needs a company’s financial statements and industry information to generate his own view about the company. Fund managers’ information and independent research firms’ information are complementary. By providing complementary information, independent research firms may be able to exist without investment banking profit because fund managers are very willing to pay for that information.

To incorporate this idea formally into the model, I assume that without indirect information, which is signal $s_I$, an information-producing agent’s own signal, has precision $rv, r \leq 1$, but with signal $s_I$, his own signal has precision $v$. This assumption captures the idea that information-producing agents need the basic information to produce their own signals. The smaller $r$, the poorer the quality of the information-producing agents’ signals without $s_I$, the more important $s_I$ is to other agents’ information production. Thus $r$ measures the complementarity between $s_I$ and other signals. Because $s_I$ is the basic and factual information such as company news, I assume that if two or more information-producing agents produce $s_I$, they obtain the same signal. I further assume that $s_I$ cannot be traded on directly, but in order to generate tradable information, such as an estimate of a company’s value, information-producing agents need to analyze and interpret the indirect information.
The model is modified as follows. At date 0, an information-producing agent can choose to be a fund manager, an analyst, or an independent analyst. At date 1, an analyst sells his information to the fund managers, and an independent analyst sells his information to both fund managers and analysts. In both cases, the information sellers make offers to the buyers as before. The events at dates 2 and 3 remain the same.

Proposition 15 shows that if $s_I$ is important enough to the information production of other agents, then even without investment banking profit, there remains an equilibrium in which there is one independent analyst:

**Proposition 15** If there is no investment banking profit for the information-producing agents, then there exists a $r > 0$, such that for $r \in (0, r)$, there exists an equilibrium in which:

(i) There is one independent analyst.

(ii) There are $N - 1$ fund managers but no analyst.

(iii) The independent analyst sells $s_I$ to all the fund managers, and the fund managers trade in the financial market as characterized by Proposition 1.\(^{22}\)

A fund manager without $s_I$ is at a disadvantage in trading in the financial market because he has poor-quality information, while other fund managers have high-quality information. The smaller $r$, the greater the disadvantage, and thus the more a fund manager is willing to pay for $s_I$. Therefore as $r$ decreases, an independent analyst’s profit from selling information increases, while his profit decreases if he chooses to be a fund manager. In the extreme case where $r = 0$, it is strictly better for the information-producing agent to be an independent analyst, given that the profit is zero if he chooses to be a fund manager while the profit is positive if he chooses to be an independent analyst. Because agents’ profits are continuous in $r$, one independent analyst may exist in equilibrium when $r$ is small enough.

\(^{22}\)Because $s_I$ is not tradable, proposition 1 applies here with $m = 0$ and $n = N - 1$. 
6 Implications

This study has a variety of implications for firms operating in the capital markets, not to mention their clients and their regulators.

6.1 Equities Research

The equities research departments at major securities firms (analysts in my model) have long been bundled with investment banking. Unfortunately, this structure has given rise to the conflict of interest problem with regard to sell-side research. From a regulator’s point of view, however, it may actually be desirable to maintain a certain level of sell-side research, since sell-side research may improve information efficiency in the economy. My analysis suggests that regulators may have to trade off the conflict of interest problem (if bundling research with investment banking necessarily cause such a problem) and the positive functions (such as improving information efficiency) of sell-side research. The global settlement reached by the SEC and New York Attorney General Elliot Spitzer with ten of the most prestigious sell-side firms seems to be consistent with my theory. That is, (i) the settlement doesn’t require the investment banks to spin off their research departments; and (ii) in an effort to promote independent research, the settlement requires the ten firms to subsidize the independent research firms.\(^23\) The intention is to let the investment banks subsidize research without undesired influence by the investment bankers.\(^24\)

\(^23\)The $1.5 billion settlement requires the sell-side firms to pay $460 million for independent research over five years, and to distribute independent research reports together with their own reports. See “Pain of Wall Street Settlement To Be Eased by U.S. Taxpayers,” Wall Street Journal, Feb. 13, 2003, for details of the settlement.

\(^24\)Solving the conflict of interest problem through direct government intervention has its own problems. For example, providing the right incentive to the chosen independent research firms can be very difficult, and such difficulty may lead to inefficiency. It is not clear whether the proposed subsidy is optimal.
6.2 Credit Rating

In the credit rating industry, the bulk of credit rating agency revenue comes from bond issuer fees rather than selling research to institutional investors.\textsuperscript{25} If one thinks of credit rating agencies and institutional investors in the bond market as analysts and fund managers in my model, respectively, this fee structure is not too difficult to understand. Just as the sell-side research departments need the investment banking subsidy, the credit rating agencies need payment from bond issuers to sustain their research function. Credit rating agencies and sell-side research departments are similar; both generate and distribute information about issuers, and both are compensated mainly by the issuers, in the form of direct payment in the former case and investment banking profit in the latter case.\textsuperscript{26} Precisely because of this similarity, the credit rating agencies are subject to the same conflict of interest problem as the sell-side equities research departments.\textsuperscript{27} As in the sell-side research, eliminating the conflict of interest problem by prohibiting credit rating agencies from receiving payment from issuers may not be a good idea. Because the credit rating agencies can make more profit by becoming bond funds, regulation prohibiting issuer fees may cause rating agencies to cease to exist as information providers.\textsuperscript{28}

Interestingly, credit rating agencies initially provided rating services free to issuers, and financed their operations solely through the sale of publications and related materials. It was not until the late 1960s and the early 1970s that rating agencies started to charge issuers for ratings. Cantor and Packer (1995) and White (2001) argue that the change was caused by the wide-spreading of copy machine, which began to make it hard for the rating agencies to maintain their profit margins. I think this explanation is inadequate for at least two reasons. First, information providers such as Dun & Bradstreet Credit Rating have been successful in selling information despite the fact that

\textsuperscript{25}According to Moody’s annual report, in 1999 close to 90\% of its revenue was from rating services, paid by the issuers.

\textsuperscript{26}There are firms that even pay research firms for coverage. For details see “Amid Shrinking Research Pool, Companies Buy Their Coverage,” Wall Street Journal, March 26, 2003.

\textsuperscript{27}As evidenced by the Nov. 15, 2002, SEC public hearing on credit rating agencies.

\textsuperscript{28}To be more precise, here I mean direct information providers. Indirect information providers may still exist, for the same reason that independent research firms do.
they are under the same pressure from cheap copying technology. In fact, when Moody’s started to charge the issuers, Dun & Bradstreet Credit Rating, the parent company of Moody’s at that time, continued to maintain itself on subscriber fees. Second, in explaining fee structure, during the SEC hearing, credit rating industry representatives never mentioned the difficulty of establishing intellectual property rights as one factor in its fee structure decisions.

My analysis suggests a more plausible explanation. Before the change in fee structure, today’s widespread professionalization of portfolio management was not well developed. Trading on one’s own wealth wouldn’t be particularly profitable simply because of limited capital. Therefore, selling information to more investors would be a better alternative for the informed. Then, as professional portfolio management became more advanced at the end of the 1960s and the early 1970s, establishing a fund became increasingly attractive to the informed. Facing increasing opportunity costs, credit rating agencies had to charge these issuers to maintain their business. As Leo O’Neill, president of Standard & Poor’s, put it during the SEC credit rating agency hearing in 2002: “The practice (charging issuers for rating services) was implemented because of increasing costs related to credit ratings surveillance and growing need for more rating coverage. Prior to that, Standard & Poor’s provided its credit rating services on the basis of subscription fees, which were not adequate to offset the increased costs of maintaining a high level of quality in this business.”

7 Conclusion

My analysis examines the allocation of information production between the buy-side and the sell-side when identical information-producing agents can choose to be either sell-side analysts or buy-side fund managers. Analysts profit by selling information to fund managers and doing investment banking; fund managers profit by trading on their own information and the information they buy from the analysts. I show that, in order for analysts to be in equilibrium, an investment bank-

\[\text{29Dun & Bradstreet provides credit information on firms around the world (indirect information).}\]

\[\text{30In the model, the assumption that the fund managers can take arbitrary large positions is more appropriate for mutual funds or hedge funds. For small individual investors, however, this assumption is unrealistic.}\]
ing subsidy to analysts is necessary, especially when there are many information-producing agents. Without subsidy, an information-producing agent earns less as an analyst than as a fund manager, because both the competition among analysts to sell information and the competition among fund managers to trade on information limit an analyst’s profits from selling information. Tying investment banking to sell-side research enables such a subsidy. I also show that it is desirable to have a positive level of analysts; as they improve social welfare by enhancing the information efficiency of the financial markets. Therefore, a total separation of investment banking from sell-side research to solve the conflict of interest problem may not be a good idea. Rather, regulators may have to trade off between the conflict of interests problem and the positive functions of sell-side research. My analysis also explains the existence of independent research and the fee structure in the credit rating industry.
8 Appendix

Proof of Proposition 1: Substituting equations (6) and (3) into fund manager $i$’s objective function (2) and simplifying, we get

$$\max_{x_i} x_i(E[\delta|F_i] - \alpha \eta(n-1)s_p - \beta \eta(n-1)E[\delta|F_i] - \eta x_i)$$  \hspace{1cm} (43)

where $E[\delta|F_i] = \frac{mv_{sp} + vs_i}{1 + mv + v}$ by Bayes theorem. The first order condition for this problem is

$$x_i^* = \frac{[1 - \beta \eta(n-1)]E[\delta|F_i] - \alpha \eta(n-1)s_p}{2\eta}$$

$$= \frac{1}{2\eta} \left\{ [1 - \beta \eta(n-1)] \frac{mv}{1 + mv + v} - \alpha \eta(n-1)s_p + [1 - \beta \eta(n-1)] \frac{v}{1 + mv + v} s_i \right\}$$  \hspace{1cm} (44)

Because $x_i = \alpha s_p + \beta s_i$ in symmetric equilibrium:

$$\alpha = \frac{1}{2\eta} [(1 - \beta \eta(n-1)) \frac{mv}{1 + mv + v} - \alpha \eta(n-1)],$$  \hspace{1cm} (45)

$$\beta = \frac{1}{2\eta} (1 - \beta \eta(n-1)) \frac{v}{1 + mv + v}.$$  \hspace{1cm} (46)

Solving equations (45) and (46), we get $\alpha$ and $\beta$. For part (ii), fund manager $i$’s expected trading profit is

$$E\left[\frac{[(1 - \beta \eta(n-1))E[\delta|F_i] - \alpha \eta(n-1)s_p]^2}{4\eta}\right].$$  \hspace{1cm} (47)

Simplifying by using the fact that $E[(E[\delta|F_i])^2] = \frac{v + mv}{1 + mv + v}$, $E[(E[\delta|F_i])s_p] = 1$, and $E[s_p^2] = 1 + \frac{1}{mv}$, we get

$$\frac{1}{4\eta} \left[ (1 - \eta \beta(n-1))^2 \frac{v + mv}{1 + v + vm} + [\alpha \eta(n-1)]^2 \frac{1 + vm}{vm} - 2[1 - \eta \beta(n-1)][\alpha \eta(n-1)] \right].$$  \hspace{1cm} (48)

Substituting in $\alpha$ and $\beta$, we get (9). For part (iii), substituting $\alpha$ and $\beta$ into (3) yields the result.
For part (iv), the market maker sets the price according to (4). But,

$$E[\delta|y] = \frac{\frac{1}{n(\alpha+\beta)}y}{1 + \left(\frac{\alpha}{\alpha+\beta}\right)^2 \frac{1}{mv} + \frac{n\beta^2}{(n(\alpha+\beta))^2v} + \frac{\sigma^2}{(n(\alpha+\beta))^2}}$$

(49)

which implies

$$\eta = \frac{\frac{1}{n(\alpha+\beta)}}{1 + \left(\frac{\alpha}{\alpha+\beta}\right)^2 \frac{1}{mv} + \frac{n\beta^2}{(n(\alpha+\beta))^2v} + \frac{\sigma^2}{(n(\alpha+\beta))^2}}$$

(50)

Together with equations (45) and (46), solving for $\alpha$, $\beta$, and $\eta$ gives the desired results.

**Proof of Corollary 2:** Proof of Part (i), (ii), and (iii) are already given. For part (iv), the volume generated by sell-side analyst $j$ and by fund manager $i$ are $naE[|s_j|]$ and $\beta E[|s_i|]$, respectively. Because $\beta > na$, as shown in part (i), and $s_i$ and $s_j$ are identically distributed, we conclude that $naE[|s_j|] > \beta E[|s_i|]$.

**Proof of Lemma 3:** Suppose analyst $q$ successfully sells to only a strict subset of the fund managers, $F_q^a$ at prices $p(F_q^a)$. Now we claim that analyst $q$ can do better by deviating to sell his information to one more fund manager, say, $i$, to whom the analyst doesn’t successfully sell in equilibrium, and keeping the offers to fund managers in $F_q^a$ the same. Analyst $q$ would be strictly better off if fund manager $i$ is willing to pay a positive price, since on top of his equilibrium payoff he receives payment from fund manager $i$. Thus we need to show only that fund manager $i$’s willingness to pay is positive.

Suppose fund manager $i$’s willingness to pay is zero; i.e., he will not benefit from knowing $s_q$. Without $s_q$, fund manager $i$ solve:

$$Max_{x_i} E[x_i(V + \delta - (V + \eta y))|F_i].$$
That is:

\[
\pi(F_i) = \max_{x_i} E[x_i(\delta - \eta(\sum_{j \neq i} x_j + x_i + z))|F_i],
\]

\[
= x_i(E[\delta|F_i] - \eta(\sum_{j \neq i} E[x_j|F_i] + x_i)).
\]

Foc of the problem is

\[
x_i(F_i) = \frac{E[\delta|F_i] - \eta \sum_{j \neq i} E[x_j|F_i]}{2\eta}.
\] (51)

Further, soc requires that \(\eta \geq 0\), which is satisfied by assumption. Also, if soc is satisfied, the objective function is quadratic, which means foc gives the unique optimum.

With \(s_q\), by the same analysis, we know the unique optimal order strategy is

\[
x_i(F_i, s_q) = \frac{E[\delta|F_i, s_q] - \eta \sum_{j \neq i} E[x_j|F_i, s_q]}{2\eta}.
\] (52)

If fund manager \(i\) doesn’t benefit from \(s_q\), \(x_i(F_i) = x_i(F_i, s_q)\) with probability 1. Otherwise, the expected trading profits conditioning on \(F_i\) and \(s_q\) have to satisfy \(\pi(x_i(F_i), F_i, s_q) < \pi(x_i(F_i, s_q), F_i, s_q)\) whenever \(x_i(F_i) \neq x_i(F_i, s_q)\) by the uniqueness of \(x_i(F_i, s_q)\). Further, we know \(\pi(x_i(F_i), F_i, s_q) \leq \pi(x_i(F_i, s_q), F_i, s_q)\), \(\forall F_i, s_q\), because fund manager \(i\) can always adopt trading strategy \(x_i(F_i)\) when it knows \(F_i\) and \(s_q\). Therefore:

\[
E[\pi(x_i(F_i), F_i, s_q)] < E[\pi(x_i(F_i, s_q), F_i, s_q)].
\]

But this says the ex ante profit of fund manager \(i\) with \(s_q\) is strictly better than without \(s_j\), which contradicts our assumption.

Therefore, with probability 1,

\[
E[\delta - \eta \sum_{j \neq i} x_j|F_i] = E[\delta - \eta \sum_{j \neq i} x_j|F_i, s_q].
\] (53)
But because \( x_i(F_i) = x_i(F_i, s_q) \) almost surely, (53) can be rewritten as

\[
E[\delta - \eta \sum_{j=1}^{n} x_j|F_i] = E[\delta - \eta \sum_{j=1}^{n} x_j|F_i, s_q].
\]  

(54)

In a linear symmetric equilibrium in which fund managers only buy information from \( k \) analysts where \( k < m \), \( x_i(F_i) = \beta s_i + a(\sum_1^k s_i^j), \forall i \). \( s_i^j \) denotes the signal of analysts \( l' \) who sells information to fund manager \( i \). Then we have

\[
\sum_{j=1}^{n} x_j = \beta \sum_{j=1}^{n} s_j + a \sum_{j=1}^{n} \sum_{l=1}^{k} s_i^j
\]

\[
= \beta \sum_{j=1}^{n} s_j + a \frac{n_k}{m} \sum_{l=1}^{m} s_l.
\]  

(55)

The second equality follows because the fund managers receive a total of \( nk \) signals. Therefore each analyst’s signal will be used by \( \frac{nk}{m} \) fund managers by symmetry. \( s_l \) denotes analyst \( l’ \)’s signal.

Substituting (55) into (54), the LHS becomes

\[
E[\delta|F_i] - \eta(\beta s_i + a \frac{n_k}{m}(\sum_1^k s_i^j)) - \eta(\beta \sum_{j \neq i} E[s_j|F_i] + a \frac{n_k}{m} \sum_{s_l \notin F_i} E[s_l|F_i])
\]

\[
= E[\delta|F_i] - \eta[\beta s_i + a \frac{n_k}{m}(\sum_1^k s_i^j)] - \eta(\beta(n-1)E[\delta|F_i] + a \frac{n_k}{m} (m-k)E[\delta|F_i])
\]

\[
= [1 - \eta[\beta(n-1) + a \frac{n_k}{m} (m-k)]] E[\delta|F_i] - \eta[\beta s_i + a \frac{n_k}{m}(\sum_1^k s_i^j)].
\]  

(56)

The second equality follows because \( \forall s_j \notin F_i, E[s_j|F_i] = E[\delta + \varepsilon_j|F_i] = E[\delta|F_i] \). Similarly, the RHS of (54) is:

\[
[1 - \eta[\beta(n-1) + a \frac{n_k}{m} (m-k-1)]] E[\delta|F_i, s_q] - \eta[\beta s_i + a \frac{n_k}{m}(\sum_1^k s_i^j)] - \eta a \frac{n_k}{m} s_q.
\]  

(57)
Substituting

\[
E[\delta|F_i] = \frac{v(s_i + \sum_1^k s_i)}{1 + (1+k)v}
\]
\[
E[\delta|F_i, s_q] = \frac{v(s_i + s_q + \sum_1^k s_i)}{1 + (2+k)v}
\]

into (54), because (54) holds for any realization of \( F_i \) and \( s_q \), and matching the coefficients, we must have

\[
[1 - \eta[\beta(n-1) + a \frac{nk}{m}(m-k)]] \frac{v}{1 + (1+k)v} = [1 - \eta[\beta(n-1) + a \frac{nk}{m}(m-k-1)]] \frac{v}{1 + (2+k)v},
\]

(58)

and

\[
[1 - \eta[\beta(n-1) + a \frac{nk}{m}(m-k-1)]] \frac{v}{1 + (2+k)v} - \eta a \frac{nk}{m} = 0.
\]

(59)

Substituting (59) into (57), we get

\[
\eta a \frac{nk}{m}(s_i + \sum_1^k s_i) - \eta[\beta s_i + a \frac{nk}{m}(\sum_1^k s_i)]
\]

\[
= \eta(a \frac{nk}{m} - \beta)s_i.
\]

(60)

Thus

\[
\eta[\beta s_i + a(\sum_1^k s_i)] = \eta x_i(F_i) = \eta x_i(F_i, s_q) = E[\delta - \eta \sum_{j=1}^n x_j|F_i, s_q]
\]

\[
= \eta(a \frac{nk}{m} - \beta)s_i.
\]

(62)

The third equality follows from (52), and the last equality follows because of (61). Matching coefficients of (62), we must have \( a = 0 \) and \( \beta = 0 \). But these two equalities make (58) impossible to hold. Therefore there is a contradiction.
Proof of Proposition 4: Conditioning on all other fund managers buying from all analysts and trading as specified in Proposition 1, fund manager $i$’s objective is

$$\max_{x_i} \quad x_i (E[\delta|F_i] - \alpha \eta (n-1) E[s_p|F_i] - \beta \eta (n-1) E[\delta|F_i] - \eta x_i)$$

where $\alpha$ and $\beta$ are as in Proposition 1. We have

$$E[s_p|F_i] = \frac{1}{m} \left( \sum_{j \in A^i} s_j + \sum_{k \notin A^i} E[s_k|F_i] \right)$$

$$= \frac{1}{m} s_l + \frac{m-l}{m} E[\delta|F_i].$$

The second equality follows because

$$E[s_k|F_i] = E[\delta|F_i].$$

Substituting (65) into the objective function and taking the first order condition, we get fund manager $i$’s optimal trading strategy

$$x^*_i(A^i) = \left( 1 - \beta \eta (n-1) - \alpha \eta (n-1) \frac{m-l}{m} \right) E[\delta|F_i] - \alpha \eta (n-1) \frac{l}{m} s_l$$

The expected profit under the optimal trading strategy is

$$\pi_i(A^i) = \frac{1}{4\eta} E[(1 - \beta \eta (n-1) - \alpha \eta (n-1) \frac{m-l}{m}) E[\delta|F_i] - \alpha \eta (n-1) \frac{l}{m} s_l]^2$$

Substituting $E[\delta|F_i] = \frac{\nu + \nu_a}{1 + \nu + \nu_b}$ into equations (67) and (68), and simplifying using $E[(E[\delta|F_i])^2] = \frac{\nu + \nu_a}{1 + \nu + \nu_b} E[(E[\delta|F_i]) s_l] = 1$, and $E[s_l^2] = 1 + \frac{1}{\nu}$, we get the desired results.
Proof of Corollary 5:

\[
\frac{d\pi_i(l)}{dl} = \frac{16(1 + v + mv)^2}{(1 + n)^2(2 + (1 + 2m + n)v)^2(1 + v + lv)^2} > 0,
\]

\[
\frac{d^2\pi_i(l)}{dl^2} = -\frac{32v(1 + v + mv)^2}{(1 + n)^2(2 + (1 + 2m + n)v)^2(1 + v + lv)^3} < 0.
\]

Proof of Proposition 8: After some simplification, we have

\[
\pi_s(m) - \pi_b(m - 1) =
\]

\[
-\frac{4v}{(1+N-m)^4} \left[ \frac{4(m-1)^2}{1+(m-1)v} + \frac{(N-m-1)(1+N-m)(5+2N+N^2-2(3+N)m+m^2)}{(2+(N+m-1)v)^2} \right]
\]

\[
+ \frac{(N+m-1)(5+2N+N^2-2(3+N)m+m^2)}{2+(N+m-1)v} - \frac{4(N-m-1)(1+N-m)(N-m)}{(2+(N+m-1)v)^2}
\]

\[
+ \frac{4(N-m)m}{1+mv} - \frac{4(1+N+m)(N-m)}{2+(N+m-1)v}.
\]

Thus \(\pi_s(m) - \pi_b(m - 1)\) has the opposite sign of the term in the square brackets. We then put together the terms in the square brackets into a big fraction. Because the denominator is positive, we need to show only that the numerator is positive. The numerator is \((1+N-m)^2\) times:

\[
B(N,m) \equiv 4(1+N-m)^2 + 4[5 + N^3 + N^2(m-1) - K(m+1)(5m-3) + m(m(7+3m)-7)]v
\]

\[
+ [13 + 24N - 6N^2 + N^4 + 8(3 + N + N^2 + N^3)m - 2(45 + N(3N-8))m^2 - 8(2N-5)m^3
\]

\[
+ 13m^4]v^2 + [-8(N-1)^2N + 2(21 + N(4 + N(-6 + N(N+4))))m + 4(1 + N(-7 + N(7 + N)))m^2
\]

\[
- 8(10 + (-4 + N)N)m^3 - 4(-7 + N)m^4 + 6m^5]v^3 + [-(-1 + N)^4
\]

\[
+ (-1 + N)(N+1)(3 + (-12 + N)N)m + (23 + N(-8 + N(-12 + N(12 + N))))m^2
\]

\[
+ 2(-3 + N(-14 + 11N))m^3 - (23 + 2(-10 + N)N)m^4 + 9m^5 + m^6]v^4 +
\]

\[-(-1 + m)m(-1 + N + m)^2(1 + N + m)^2v^5.
\]
We show that $B > 0$ in three steps: i) $B$ is convex in $N$; (ii) $B$ is increasing in $N$; and (iii) $B$ is positive. First

\[
\frac{\partial^2 B}{\partial N^2} = 4\{2 + 2(-1 + 3N + m)v + [-3 + 3N^2 + 12Nm + (4 - 3m)m]v^2 \\
+ 2[4 + 3N^2m + m(-3 + (7 - 2m)m) + 3N(-2 + m(2 + m))]v^3 \\
+ [-3 + m - m^2(6 + (-11 + m)m) + 6N(1 + 3(-1 + m)m) + 3N^2(-1 + m + m^2)]v^4 \\
+ (-1 + m)m(-1 + 3(N + m)^2)v^5\}. \tag{69}
\]

The coefficients of $v$, $v^2$, $v^3$, and $v^5$ is trivially non-negative for $m \geq 1$ and $N \geq m + 1$. The coefficient of $v^4$ is clearly increasing in $N$. So, for a given $m$, the smallest possible value of the coefficient can be achieved by the smallest $N$, which is $m + 1$. Substituting $N = m + 1$ into the coefficient, the coefficient of $v^4$ becomes

\[2m[-7 + m^2(19 + m)], \tag{70}\]

which is clearly positive for $m \geq 1$. Thus we conclude that the coefficient of $v^4$ is positive. Hence, we have shown $\frac{\partial^2 B}{\partial N^2} > 0$.

Because $\frac{\partial B}{\partial N}$ is increasing in $N$ for a given $m$, the smallest $\frac{\partial B}{\partial N}$ can be achieved by the smallest $N$. Again, the smallest possible $N$ is $m + 1$. At $N = m + 1$, $\frac{\partial B}{\partial N}$ is

\[\frac{\partial B}{\partial N}\Big|_{N=m+1} = 16v(1 + v)(1 + v + mv)(1 + m(1 + (-1 + m)v)(1 + v + 2mv). \tag{71}\]

Clearly, $\frac{\partial B}{\partial N}\Big|_{N=m+1} > 0$. Thus we can conclude that $\frac{\partial B}{\partial N} > 0$.

Finally, because $\frac{\partial B}{\partial N} > 0$, the smallest $B$ can be achieved by the smallest $N$. Substituting $N = m + 1$ into $B$, we get

\[B\Big|_{N=m+1} = 16v(1 + mv)^2(1 + v + mv)[2 + (-1 + m)m(1 + v + mv)], \tag{72}\]
which is positive. Thus, we have shown that for a given $m \geq 1$, $B$ is positive. Hence, $\pi_s(m) - \pi_b(m - 1)$ is negative for $1 \leq m \leq N - 1$.

Part (ii) follows directly by checking that $m^* = 0$ satisfies both (24) and (25).

**Proof of Proposition 10:** $\pi_b(m^*) > \pi_r(m^*)$ if and only if

\[
\frac{1}{4\eta} \frac{4v[(n^* + 1)^2 + [4m^2 + (1 + n^*)^2 + m^*(1 + n^*(6 + n^*))v + m^*(1 + 2m^* + n^*)^2v^2]}{(1 + n^*)^2(1 + m^*v)(2 + (1 + m^* + n^*)v)^2} > \frac{1}{4\eta} \frac{16n^*v(1 + v + m^*v)}{(1 + n^*)^2(1 + m^*v)(2 + (1 + m^* + n^*)v)^2},
\]

After some simplification, this condition is equivalent to

\[
4v[(n^* + 1)^2 + [4m^2 + (1 + n^*)^2 + m^*(1 + n^*(6 + n^*))v + m^*(1 + 2m^* + n^*)^2v^2 - 16n^*v(1 + v + m^*v)] > 0.
\]

Simplifying the right-hand side, we get

\[
4v[(-1 + n^*)^2 + [4m^2 + (-1 + n^*)^2 + m^*(1 + n^*)^2v + m^*(1 + 2m^* + n^*)^2v^2],
\]

which is clearly positive for $m^* \geq 1$ and $n^* \geq 1$.

**Proof of Proposition 11:** Straightforward algebra yields

\[
\lim_{m \to \infty} \frac{\pi_r(m, n)}{\pi_b(m, n)} = \lim_{m \to \infty} \frac{\pi_r(m, n)}{\pi_b(m - 1, n + 1)} = \lim_{m \to \infty} \frac{n}{m^2v} = 0,
\]

\[
\lim_{n \to \infty} \frac{\pi_r(m, n)}{\pi_b(m, n)} = \lim_{n \to \infty} \frac{\pi_r(m, n)}{\pi_b(m - 1, n + 1)} = \lim_{n \to \infty} \frac{4(1 + v + mv)}{n(1 + v)(1 + mv)} = 0,
\]

\[
\lim_{N \to \infty} \frac{\pi_r(xN, (1 - x)N)}{\pi_b(xN, (1 - x)N)} = \lim_{N \to \infty} \frac{\pi_r(xN, (1 - x)N)}{\pi_b(xN - 1, (1 - x)N + 1)} = \lim_{N \to \infty} \frac{4 - 4x}{N(1 - x)^2 + (1 + x)^2v} = 0,
\]

which proves part (i). Part (ii) follows directly from part (i) and (28).

Repeating the limit analysis and keeping $\Gamma \equiv Nv$ constant, we get
\[
\lim_{m \to \infty} \frac{\pi_r(m, n)}{\pi_b(m, n)} = \lim_{m \to \infty} \frac{\pi_r(m, n)}{\pi_b(m - 1, n + 1)} = \lim_{m \to \infty} \frac{n}{m \Gamma(\mu)} = 0, \quad (78)
\]
\[
\lim_{m \to \infty} \frac{\pi_r(m, n)}{\pi_b(m, n)} = \lim_{n \to \infty} \frac{\pi_r(m, n)}{\pi_b(m - 1, n + 1)} = \lim_{n \to \infty} \frac{4}{n} = 0, \quad (79)
\]
\[
\lim_{N \to \infty} \frac{\pi_r(xN, (1 - x)N)}{\pi_b(xN, (1 - x)N)} = \lim_{N \to \infty} \frac{\pi_r(xN, (1 - x)N)}{\pi_b(xN - 1, (1 - x)N + 1)} = \lim_{N \to \infty} \frac{4}{N(1 - x)} = 0, \quad (80)
\]

which is part (iii).

**The proof of Proposition 12:** It is enough to show that, in two economies, denoted by 1 and 2, if \( \Sigma_{r0} > \Sigma_{r1}, W_0 > W_1 \).

Because \( \Sigma_{r0} > \Sigma_{r1}, P_2^0 \) and \( P_2^1 + \phi \) have the same distribution, where \( \phi \sim N(0, \Sigma_{r0} - \Sigma_{r1}) \) and \( \phi \) and \( P_2^1 \) are independent. Thus we have

\[
W_0 = E[P_2^0 q^*(P_2^0) - f(q(P_2^0))] = E[(P_2^1 + \phi)q^*(P_2^1 + \phi) - f(q^*(P_2^1 + \phi))]. \quad (81)
\]

But

\[
(P_2^1 + \phi)q^*(P_2^1 + \phi) - f(q^*(P_2^1 + \phi)) \geq (P_2^1 + \phi)q^*(P_2^1) - f(q^*(P_2^1)), \quad (82)
\]

since \( q^*(P_2^1 + \phi) \) uniquely maximizes \( (P_2^1 + \phi)q - f(q) \). Furthermore, the inequality in (82) is strict for \( \phi \neq 0 \). Taking expectation of both sides, we get

\[
E[(P_2^1 + \phi)q^*(P_2^1 + \phi) - f(q^*(P_2^1 + \phi))] > E[(P_2^1 + \phi)q^*(P_2^1) - f(q^*(P_2^1))]
\]

\[
= E[E[(P_2^1 + \phi)q^*(P_2^1) - f(q^*(P_2^1))|P_2^1]]
\]

\[
= E[P_2^1 q^*(P_2^1) - f(q^*(P_2^1))]. \quad (83)
\]

The last equality follows because \( E[\phi|P_2^1] = 0 \). Combining (81) and (83) yields \( W_0 > W_1 \).
Proof of Proposition 13: We have

\[
\frac{\partial \Sigma_r}{\partial m} = \frac{v}{(1 + N - m)^3(2 + (1 + N + m)v)^3}\{4(-1 + (N - m)^2 - 2m)(1 + N - m) + 2[-2 + N^4 - N^3(3 + 2m) + N^2(-7 + 5m) - m(3 + m(9 + (-7 + m)m)) + N(-5 + m(8 + m(-9 + 2m)))v - [(1 + N)^2(1 + N(2 + 5N)) - 2(1 + k)^2(-1 + 7N)m + 12(1 + 2N(1 + N))m^2 - 2(1 + 9N)m^3 + 3m^4]v^2\}.
\]

Therefore, \(sgn\left(\frac{\partial \Sigma_r}{\partial m}\right) = sgn\{\cdot\}\). For ease of notation, we denote \(\{\cdot\} \equiv d(N, m)\). First we show part (iii).

\[
d|_{m = N - 1} = -16[-1 + N + 2(1 + (-1 + N)N)v + (1 + N - N^2 + N^3)v^2] < 0, \ \forall N \geq 2. \quad (84)
\]

Because \(\frac{\partial \Sigma_r}{\partial m}\) is continuous in \(m\), as \(m\) close to \(N, \frac{\partial \Sigma_r}{\partial m} < 0\).

For part (ii), setting \(4(-1 + (N - m)^2 - 2m) = 0\), and solve for \(m\), we get \(m = 1 + N \pm \sqrt{2(1 + N)}\). The larger root is clearly greater than \(N\). So, if \(m < 1 + N - \sqrt{2(1 + N)}, 4(-1 + (N - m)^2 - 2m) > 0\). However, as \(v \to 0, d \to 4(-1 + (N - m)^2 - 2m)(1 + N - m)\). Therefore, if \(m < 1 + N - \sqrt{2(1 + N)}\), there exists a \(v\), such that if \(v < v_\star, \frac{\partial \Sigma_r}{\partial m} > 0\).

For part (i), we first look at the coefficient of \(v^2\) of \(d\), and denote it \(-V^2\). We are going to show that \(V^2 > 0\).

\[
\frac{\partial V^2}{\partial m} = -2(1 + N)^2(-1 + 7N) + 24(1 + 2N(1 + N))m - 6(1 + 9N)m^2 + 12m^3 \quad (85)
\]

\[
\leq -2(1 + N)^2(-1 + 7N) + 24[1 + 2N(1 + N)]m - 6(3 + 7N)m^2. \quad (86)
\]

The inequality follows because \(12m^3 \leq 12m^2(N - 1)\). Denote \(V^2' \equiv -2(1 + N)^2(-1 + 7N) + 24[1 + 2N(1 + N)]m - 6(3 + 7N)m^2\) is a quadratic function of \(m\). Moreover, the determinant of \(V^2'\) is:

\[
\Delta = 45 + 120N + 66N^2 - 48N^3 - 3N^4. \quad (87)
\]
Also, $\frac{\partial \Delta}{\partial N} = -12[-10 + N[-11 + N(12 + N)]]$. Because $N \geq 2$, $\frac{\partial \Delta}{\partial N}$ is negative. However, $\Delta|_{N=3} < 0$. Therefore, $\Delta < 0$, given $N \geq 3$. That means that if $N \geq 3$, $V'2 < 0$, which in turn implies $\frac{\partial V^2}{\partial m} < 0$.

For $N = 2$, $V'2|_{N=2} = -6[39 + m(-52 + 17m)]$. Moreover, $\frac{\partial V^2}{\partial m}|_{N=2} = 312 - 204m > 0$ because $m \leq 1$. That is, the biggest $V'2|_{N=2}$ can be achieved by the largest $m$, which is 1 here. At $m = 1$, $V'2|_{N=2} = -24$. Therefore, $V'2|_{N=2} < 0$, which again implies $\frac{\partial V^2}{\partial m} < 0$. In conclusion, for $N \geq 2$, $\frac{\partial V^2}{\partial m} < 0$.

Thus the smallest $v2$ is achieved when $m = N - 1$, for given $N \geq 2$. Substituting $m = N - 1$ into $V2$:

$$V2 = 16(1 + N - N^2 + N^3) > 0.$$ (88)

Because $d$ is quadratic in $v$ and $V2 > 0$, as $v \to \infty$, $d \to \infty$. Therefore, for given $N$ and $m$, there exists a positive $\pi$, so that if $v > \pi$, $\frac{\partial \Sigma}{\partial m} < 0$.

**Proof of Proposition 14:** Calculating the limits directly yields the results.

**Proof of Proposition 15:** At date 2, along the equilibrium path, the equilibrium is characterized by Proposition 1, with $m = 0$ and $n = N - 1$. At date 1, by the same analysis as in section 3.2, the equilibrium price for an independent analyst’s information is

$$p = \frac{4(1-r)v(1+v)}{(2+v+nv)^2(1+rv)}.$$ (89)

Therefore, the independent analyst’s equilibrium profit is

$$\pi_i = \frac{4n(1-r)v(1+v)}{(2+v+nv)^2(1+rv)},$$ (90)

and a fund manager’s profit is

$$\pi_b = \frac{4rv(1+v)^2}{(1+rv)(2+v+nv)^2}.$$ (91)
let us first check the independent analyst’s IC condition. Clearly the independent analyst is not willing to be an analyst, since if he does, because $s_I$ is not produced, every information-producing agent’s signal has precision $rv$, and by Proposition 8, the analyst is better off being a fund manager. If the independent analyst decides to be a fund manager, then there will be $n + 1$ fund managers and each fund manager’s signal has precision $rv$. Thus the deviating independent analyst will get

$$\pi_b(n + 1) = \frac{4rv(1 + rv)}{(2 + rv + (n + 1)rv)^2}. \tag{92}$$

Thus, the independent analyst’s IC condition is satisfied iff $\pi_i - \pi_b(n + 1) \geq 0$. But

$$\lim_{r \to 0} [\pi_i - \pi_b(n + 1)] = \frac{4nv(1 + v)}{(2 + v + nv)^2} > 0. \tag{93}$$

Therefore, there exists an $r^* > 0$, such that if $r \in (0, r^*)$, the independent analyst’s IC is satisfied.

Now let’s check a fund manager’s IC. If a fund manager deviates to be an independent analyst, he will produce the same $s_I$ as the other independent analyst. But in the market for information, the two independent analysts will engage in Bertrand competition, which implies that the deviating fund manager’s profit is zero. Thus a fund manager is not willing to deviate to become an independent analyst. If a fund manager deviates to be an analyst, there are two cases in the market for information. The first is that the independent analyst doesn’t sell $s_I$ to the analyst; the second is that he does sell to the analyst. But because the independent analyst has all the bargaining power, the analyst has the same payoff in both cases. Let’s consider the first case, in which there are $n - 1$ fund managers whose signals have precision $v$, and 1 analyst whose signal has precision $rv$. Calculating the analyst profit using the same method as in section 3.2, we get

$$\pi_a(n - 1) = \frac{16(n - 1)rv(1 + v + rv)}{n^2(1 + v)(2 + 2rv + nv)^2}. \tag{94}$$
Thus, a fund manager’s IC is satisfied iff $\pi_b - \pi_s(n - 1) \geq 0$. Let $r \to 0$,

$$\lim_{r \to 0} \frac{\pi_b}{\pi_s(n - 1)} = \frac{n^2(1 + v)^2(2 + nv)^2}{4(n - 1)(2 + v + nv)^2}.$$  \hspace{1cm} (95)

Let $D(v, n) \equiv n^2(1 + v)^2(2 + nv)^2 - 4(n - 1)(2 + v + nv)^2$. Notice that $\lim_{r \to 0} \frac{\pi_b}{\pi_s(n - 1)} > 1$ iff $D(v, n) > 0$. To show that $D(v, n) > 0$, we first show that $\frac{\partial^2 D}{\partial v^2}(v, n) > 0$, then $\frac{\partial D}{\partial v}(v, n) > 0$, and finally $D(v, n) > 0$. After some algebra,

$$\frac{\partial^2 D}{\partial v^2}(v, n) = 2\{4 + n[4 + n^2(4 + n + 6(2 + n)v + 6nv^2)]\} > 0.$$  \hspace{1cm} (96)

So $\frac{\partial D}{\partial v}(v, n)$ is strictly increasing in $v$; i.e., $\frac{\partial D}{\partial v}(v, n) > \frac{\partial D}{\partial v}|_{v=0}$ for $v > 0$.

$$\frac{\partial D}{\partial v}|_{v=0} = 4(4 + (n - 2)n^2).$$  \hspace{1cm} (97)

Foc and soc show that the unique solution to

$$\min_{n > 0} 4(4 + (n - 2)n^2)$$  \hspace{1cm} (98)

is $n = \frac{4}{3}$, and the minimum is $11.26 > 0$. Thus, $\frac{\partial D}{\partial v}(v, n) > 0$; i.e., $D(v, n)$ is strictly increasing in $v$. But

$$D(0, n) = 4(n - 2)^2 > 0.$$  \hspace{1cm} (99)

Therefore, $D(v, n) > 0$.

Because $\lim_{r \to 0} \frac{\pi_b}{\pi_s(n - 1)} > 1$, there exists a $r^{**} > 0$ such that if $r \in (0, r^{**})$, then $\pi_b - \pi_s(n - 1) \geq 0$, i.e., the fund manager’s IC is satisfied.

The Proposition is thus proved with $r = \min\{ r^*, r^{**}\}$.
References


