Market Fragmentation Metrics

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Version 0.20: April 19, 2005
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Abstract

Markets fragment when the same instruments or instruments embodying the same risks trade simultaneously in separate markets. Fragmentation leads to increased dealer intermediation, by separating into disjoint markets traders that otherwise could have interacted directly with each other. We develop and estimate measures of market fragmentation, in an effort to better understand the deleterious effect market fragmentation may have on order execution quality. We estimate these measures using data from five U.S. option exchanges prior to the implementation of the linkage plan. Our measures involve reconstructing the limit order book of each market at the arrival time of each order, and also constructing the hypothetical order book that would have existed had all the same orders been routed to a single, consolidated book. We then identify, among all incoming orders that would have been immediately marketable in a consolidated market, the proportion that were not marketable in a fragmented market. In these cases, there are two parties potentially harmed by fragmentation: the investor behind the incoming order that is not executed and the party behind the standing quote that was not executed. For customer orders in both categories, we investigate the subsequent history to determine the proportion that were cancelled, received delayed executions, or remained unexecuted at various future points in time. We also use our method to look at fragmentation of order flow across options with different strike prices. This requires that all option orders be translated into delta-equivalent orders in a hypothetical at-the-money contract. Both fragmentation across exchanges and fragmentation across option series can have a nontrivial impact on execution quality.
Market Fragmentation Metrics

Markets fragment when instruments embodying the same risks trade in multiple locations or forms. Fragmentation may occur across multiple markets that trade identical instruments or across multiple instruments whose values depend on the same underlying risks.

Fragmentation concerns traders and regulators because it can increase transaction costs. In fragmented markets, buyers and sellers interested in trading essentially the same risks often arrive in different markets. Without fragmentation, they might easily trade with each other. In fragmented markets, they often must trade with dealers or arbitrageurs who use their inventories and hedge portfolios to intermediate between the natural buyers and sellers. Since these intermediaries must be compensated for their efforts, the costs of their intermediation are costs of fragmented markets.

The tension between fragmentation and consolidation underlies most major public policy issues in market structure. The continuing debate about what the national market system should be is a debate about whether markets should be more consolidated or less consolidated. Debates about who should have market information and market access, and whose orders and quotes should have priority are debates about the distribution of market power between dealers and investors. The outcomes of these debates have profound implications for the degree of fragmentation that will occur in our markets. Accordingly, it is important to know more about market fragmentation.

This paper defines and estimates some new measures of market fragmentation. These measures use order flows to characterize what would have happened upon the arrival of each
order if all standing limit orders and dealer quotes in multiple markets for a given underlying risk had been consolidated into a common limit order book at the moment of the arrival. Within this framework, we are able to separately measure the effects of fragmentation on the execution quality of incoming market orders, incoming limit orders, and standing limit orders.

Incoming market orders may be harmed by fragmentation if they are not routed to the market where they can obtain the best price. To measure the potential harm to market orders, we determine what prices market orders would have received had they been executed against the best limit orders and dealer quotes available in our artificially consolidated market. We then compare these prices to the prices that would have been obtained in a similar reconstruction of the fragmented market. The latter prices occasionally differ from actual market order execution prices when market makers offer price improvement. This often happens when exchange market makers match the best prices quoted elsewhere to avoid trade-throughs. To the extent that execution prices are worse in the reconstructed fragmented market than in the artificially consolidated market, the difference represents a potential cost imposed on investors. Since this measure does not include market-maker price improvements, it may be viewed as an upper estimate of the costs imposed on market orders by fragmentation. We also compare market order execution prices in our artificially consolidated markets to their actual execution prices in the real-world (fragmented) markets. Because this comparison allows for price improvement in the fragmented market, but not in the consolidated market, it should be viewed as a lower estimate.

For incoming limit orders, we classify hypothetical outcomes by whether or not the incoming order would have been marketable in our hypothetical consolidated market. For marketable orders, we consider whether the same order would have been marketable in the reconstructed fragmented market. The existence of limit orders that can be executed
immediately in a consolidated market but not in a fragmented market indicates that fragmentation may harm investors by causing their limit orders to receive delayed execution or no execution. Thus, as a measure of the potential costs of fragmentation, we compute the fraction of limit orders that are marketable in the hypothetical consolidated market but not in the reconstructed fragmented market.

Again, this fraction only measures potential costs because some orders might have been executed promptly even though they were not marketable in the fragmented market. To investigate more fully whether the investors submitting these orders were harmed, we follow the subsequent history of these orders to determine what proportion were later executed, what proportion were cancelled, and what proportion remained unexecuted. We depict these proportions graphically, as a function of time since submission.

Market fragmentation also imposes costs on investors with standing limit orders when incoming orders on the other side are routed to other venues. To measure this aspect of fragmentation, we identify the set of all incoming orders that would have been marketable in the reconstructed fragmented market, and for each such order, identify who the counterparty would have been in the fragmented market. In other words, we identify the dealer quote or standing limit order against which the incoming order should have traded, in the fragmented market. We then determine whether the counterparties would have been the same in a hypothetical market where all orders and dealer quotes are consolidated into a single book, and where price, public, and time priority are enforced. Counterparty differences indicate that fragmentation has caused consolidated market-wide priority to be violated so that the investors behind the unexecuted standing limit orders have been disadvantaged. For each such instance, we
determine whether the event represented a violation of price priority, public order precedence, or
time precedence, and we track the subsequent history of the standing order.

Our measures of fragmentation implicitly assume that all traders would have used the
same order submission strategies in consolidated markets that they actually used in fragmented
markets. Although practitioners and academics recognize that this assumption is unrealistic, it
provides a useful basis for characterizing various dimensions of fragmentation. If our purpose
were to obtain absolute measures of all costs of fragmentation, we would be much more
concerned about relying upon this assumption. Instead, we believe that our measures provide
useful information about relative degrees of fragmentation. We use our results to analyze the
determinants of fragmentation across markets, and leave it for others to estimate its absolute
costs.

Our analyses also do not value the benefits of fragmentation. Markets fragment when
traders value the conveniences that various markets or various instruments afford them.
Fragmentation across competing market centers may also generate indirect benefits if it makes
market makers quote more competitively, or creates incentives for exchanges to innovate. We
do not measure these benefits of fragmentation.

Markets may also fragment if brokers attempt to exploit their order flows. To the extent
that these agency problems lead to poorer execution, our metrics will include these costs of these
agency problems, but they will not separately identify them.

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1 Concerns about this assumption also led us to base our measures on the consolidated order book that would have
existed at the moment of arrival of each order, as opposed to the consolidated order book that would have existed
had the actual order flow been processed in a consolidated market. We recreate the limit order book upon the arrival
of each order rather than see how the limit order book would have evolved in the consolidated market.
Our measures arbitrary assume that a fully consolidated market would match orders based strictly on price priority, public order precedence, and time precedence, and that these matches would take place instantaneously when marketable orders arrive. We employ price priority and time precedence because all fully consolidated trading systems use price priority and time precedence. We include public order precedence because exchange rules or the Manning decision subject dealers in US equity markets to public order precedence. We assume instantaneous executions to mimic the performance of electronic trading systems.

We illustrate our measures by characterizing fragmentation in US listed options markets. Initially, we focus on measuring the effects of fragmentation of order flow on a given option series across the five option exchanges. Fragmentation across trading locations for identical instruments is a natural starting point. This type of fragmentation is intuitively easy to understand, and because the risk exposures are identical, no additional assumptions or transformations are needed to make the markets comparable. Option contracts with identical features negotiated on different exchanges are perfect substitutes in the US, where the exchanges share the same clearinghouse. In this respect, the US listed options market provides a natural laboratory for examining fragmentation.

Next, we apply our methods to the more difficult problem of estimating the effects of fragmentation across different instruments that embody the same risks. Option trading within a given class is fragmented across strike prices, across maturities, and across calls and puts. Our goal is to build a single book that consolidates all orders to trade the same underlying risks in a meaningful way that does not depend on which options contract the trader used to express the order. This task is complicated by the fact that option values depend on both the level and the volatility of the underlying stock. Consequently, the most appropriate way to measure the costs
of fragmentation will depend on the objectives of the investor. An investor who is buying a call option in order to take a bullish position in the stock might view selling a put option as a substitute vehicle. On the other hand, an investor who is buying a call option in order to buy volatility would view buying a put option as a substitute.

Thus, building a consolidated order book that incorporates all options and that can serve as a basis for a meaningful measure of fragmentation costs is inherently difficult. Rather than attempt to do so, we limit our attention to fragmentation across options with different strike prices and common maturities since both directional and volatility traders consider them substitute instruments. To ensure that they are reasonably close substitutes, we restrict our attention only to fragmentation across strike prices that are close to the money.

To analyze fragmentation across series within a given class, we must first translate all orders in the various series into an equivalent representation so that we can construct a common limit order book. We do this by expressing all order prices and sizes as their delta equivalents in the underlying security. Our characterization of fragmentation depends critically on the option-pricing model that we use to do the translation. For example, if the model fails to account for volatility smile, how far the translated order prices are from the market will depend on their strike prices. We use the Black-Scholes formula, calibrated to empirically estimated volatility smiles. This choice is motivated by the observation that option market makers typically use a volatility smile approach for automatically updating their quotes, and also by research by Dumas, Fleming, and Whaley (1998) indicating that this approach provides a reasonably accurate forecast of option price changes.

Also, our choice to translate orders to a delta-equivalent market reflects an implicit assumption that traders only value delta exposure. However, vega and gamma exposures also
concern many traders. An alternative approach would have been to assume that investors care only about volatility exposure, and translate orders into gamma- or vega-equivalent delta-neutral straddles. Because we restrict our attention to fragmentation across strike prices, we only consider interactions between orders with opposite exposures to both delta and volatility. If we repeated our analysis in a gamma-equivalent market, this would change the relative aggressiveness of different orders in the consolidated market, but it would preserve the set of orders on each side of the market. Therefore, we believe that our results would be essentially similar in a vega-translated market. The choice of translation is likely to have a more important impact on measures of fragmentation across calls and puts.

Our examination of fragmentation across strike prices within a given class has important regulatory implications for the question of how many strike prices markets should introduce within a given class. The greater the number of strike prices within a class, the more important is the role of intermediation. Increasing the number of strike prices within a series changes the character of trading from that of an order driven market to that of quote driven market.

We summarize the prior literature on fragmentation in section 1. We describe our data set in section 2, and introduce our methods in Section 3. In section 4, we present characterizations of fragmentation across exchanges, and then extend the method to characterize fragmentation across strike prices in Section 5. The paper closes in Section 6 with a discussion of implications of our results for regulatory policy.

1. Literature Review

Numerous authors have studied the effects of market fragmentation. Harris (1993, 2002) describes in general terms the economic forces causing markets to fragment or consolidate. On
the theoretical front, Biais (1993) uses a rational expectations framework to compare the behavior of liquidity suppliers with random inventory endowments in a centralized exchange (where market makers observe their competitors’ quotes) and a pure dealership market (where they do not). His analysis shows how fragmentation can affect the equilibrium supply of liquidity when it decreases the degree of transparency across dealers. His model predicts that the variability of the bid ask spread is greater in centralized markets, but the average spread should be the same in centralized and fragmented markets. De Frutos and Manzano (2002) show that when the dealers in the Biais (1993) model are risk-averse, expected spreads are actually narrower in fragmented markets, but Yin (2005) shows that this result reverses, and fragmented markets have wider spreads, when search costs are added to the model.

In these models, the key feature differentiating centralized and fragmented markets is the degree of transparency—specifically, the extent to which dealers can observe each others’ quotes, and the degree to which it is costly for customers to search for the best quote. In the current paper, we analyze fragmentation across markets that publicly disseminate current quotes. In this context, fragmentation is not an issue of transparency, but of the extent to which the market structure permits and encourages customer order interaction. We would expect the primary effects of fragmentation to result from the effective undermining of priority rules, rather than a decrease in transparency.

On the empirical side, several authors have looked at fragmentation in NYSE-listed stocks across the NYSE, the regional exchanges, and Nasdaq. Early work in this area, such as the studies of Branch and Freed (1977) and Hamilton (1979) examined whether bid ask spreads were correlated with off-exchange trading activity. Others authors, such as Christie and Huang (1994), Barclay (1997) and Bennett and Wei (2003), have analyzed the effects of fragmentation by
comparing market quality statistics before and after stocks move from a more fragmented to a less fragmented structure (e.g. stocks that switch from Nasdaq to NYSE). These studies consistently find that spreads decline when stocks move to a less-fragmented structure.

Various authors have examined market share, execution quality, and price impact measures in fragmented markets, and have addressed the question of whether payment for order flow can be used as a device for “cream skimming”, or siphoning off profitable (uninformed) order flow from the primary market. For example, Blume and Goldstein (1997) and Bessembinder (2003) find that trading venues can attract substantial additional market share by quoting at the inside market, but that markets attract trading volume even when they are not quoting at the inside market, suggesting that investors’ orders are not always routed to the best market. Bessembinder’s (2003) evidence in particular suggests that investors are being harmed as a result of their orders being routed to markets with inferior prices.

Others have more specifically focused on the relation between fragmentation, payment for order flow, and informed trading. Easley, Kiefer, and O’Hara (1996) find that trades executed on the Cincinnati Stock Exchange are less informative than those executed on the New York Stock Exchange, consistent with hypothesis that payment for order flow can be used as an effective device for fragmenting order flow into less-informed and more-informed channels. Bessembinder and Kauffman (1997) also find evidence that trades executed off the exchange are less informative than those executed on the NYSE. On the other hand, Battalio (1997) finds that the establishment of payment for order flow by Madoff in 1988-1990 was associated with a narrowing of the bid-ask spread on the NYSE, suggesting that the effects of increased competition more than offset any increase in adverse selection costs faced by the NYSE specialist.
A related literature addresses specifically the practice of order preferencing, a form of fragmentation. Dutta and Madhavan (1997) show in a game-theoretic model that order preferencing and price-matching arrangements can in equilibrium lead to higher transaction costs for investors. Chung, Chuwonganant, and McCormick (2004) provide empirical evidence from the NASDAQ market in support of this prediction, in their study of the cross-sectional relation between fragmentation and transaction costs. In contrast, Battalio, Greene, and Jennings (1997) study the effects of increased fragmentation in the listed market as a result of the Cincinnati Stock Exchange’s “Preferencing Dealer Program” and the Boston Stock Exchange’s “Competing Specialist Initiative,” and find that these events did not adversely affect trading costs in the listed market. Moreover, Peterson and Sirri (2003) show that the market quality statistics, such as effective spreads and limit order execution rates, are superior for the regional exchanges that have preferencing programs than for the other regional exchanges.

Our approach in the current paper involves determining the fate of customer orders in a simulated world where same orders are submitted under an alternative market structure. In this respect, our method resembles that used by Battalio, Greene, Hatch, and Jennings (2002) in their analysis of order routing decisions in the equity market. These authors examine a sample of limit orders submitted to regional markets and reconstruct the execution the same orders had they been submitted to the New York Stock Exchange.

Finally, various other authors have shown that investors benefit from competition across markets. To the extent that fragmentation promotes competition or inhibits the development of non-competitive market structures, it can help improve market quality. In the context of option markets, Mayhew (2002), Battalio, Hatch, and Jennings (2004), and DeFontnouvelle, Fishe, and
Harris (2004) provide evidence of the benefits of competition across exchanges. In the current paper, our goal is not to address the benefits of competition, or to weigh the costs and benefits of fragmentation. Rather, our focus is on measuring the costs of fragmentation within a competitive market.

2. Data

Our study is based on the system order data reported by US option exchanges in their Consolidated Options Audit Trail System (COATS) files. Our sample consists of three weeks of COATS data, covering fourteen trading days from January 6, 2003 to January 24, 2003, from all five domestic option exchanges—the American Stock Exchange, Chicago Board Options Exchange, International Securities Exchange, Pacific Exchange, and Philadelphia Stock Exchange. This data set contains detailed information about all electronic orders submitted to the exchanges including time-stamped records of order arrivals, executions, and cancellations. It does not include non-system orders (for example, spread orders and orders that were telephoned to the floor of the exchange).

As our goal is to study fragmentation, we focus on equity options that are listed on all five exchanges. Using OPRA quotes, we identified 451 underlying securities with options quoted on all five exchanges on January 6, 2003, the first day of our sample. After excluding options on exchange-traded funds, we were left with 444 underlying stocks. No options orders were reported during our three-week sample for two of these stocks and only one order was reported for another stock. We excluded these stocks and based our analysis on the remaining 441 names.

As an initial measure of the degree of fragmentation across exchanges, we compute the Hirfendahl index, or sum of squared market shares, based on the number of orders submitted for
each of the 441 classes in our sample. A value of 1.0 indicates that all orders are sent to a single
exchange, and a value of 0.2 indicates a perfectly equal distribution across five exchanges. For
the classes in our sample, the Hirfendahl index ranges from 0.21 to 0.88, with a mean of 0.33 and
a median of 0.31. Of the 441 classes in our sample, 413 classes had orders submitted to all five
exchanges, 24 had orders submitted to only four exchanges, and 4 had orders submitted to only
three exchanges.

OPRA reported market data for 602,762 series-days in these 441 stocks during our three-
week sample period. We analyze only a subset of these series-days that passed several filters.
First, we require that the underlying stock price on the previous day be above $2.50 and
eliminate any series-day for which no public order was present any time within the day. Only
227,325 series-days pass these two filters, mostly due to the second filter.

For each underlying stock, we only examine options at the two closest expiration dates.
Our sample period contains one expiration date at the end of the second week. Thus, for the first
two weeks of our sample, we examine at options expiring on January 18 and February 22, and
for the final week in our sample, we examine at options expiring on February 22 and March 22.
For most classes, the trading volumes and open interest in contracts that mature in the more
distant future are very low (Table 1). Since traders show little interest in these series, we do not
expect that examining them will provide much useful information about fragmentation.

For each day in the sample, we examine only those series with strike prices that were near
the money. Deep-in- and deep-out-of-the-money options generally are much less actively traded
than near-the-money options (Table 1). Although we might learn more about fragmentation
from the deep-in and deep-out options, the delta equivalent translations of deep-out-of-the
money options and the vega equivalent translations of deep-in-the money options are highly
sensitive to model assumptions. We exclude these series to ensure that our results primarily reflect information about fragmentation and not inadequacies in our translation procedures. Specifically, when the underlying stock price is above $50, we look at all strikes within 10 percent of the previous day’s underlying stock closing price. For stock prices between $25 and $50, we include strikes within $5.00 of the previous day’s close, and when the stock price is below $25, we include strikes within $2.50 of the previous day’s close. We eliminate a small number of series-days for which the previous day’s closing price was below $2.50. These definitions are necessary to guarantee that our “at-the-money” range always includes at least two strike prices.

The application of these filters yields a final sample size of 47,944 series-days, corresponding to 7,517 different option series over our 14-day sample. Table 1 summarizes the effect of these filters on the final sample.

For these series-days, the COATS orders files reported by the five option exchanges contain records of 1,334,722 system orders. Across the 47,944 series-days, the number of orders in our sample ranges from 1 to 3,463, with a mean of 28 and a median of 9. Across the 7,517 unique series, the number of orders ranges from 1 to 11,770, with a mean of 177 and a median of 40. Across the 441 classes, the number of orders ranges from a low of 34 to a high of 52,586, with a mean of 3,058 and a median of 1,330.

As part of our analysis, we will need to identify the current bid-ask quote on each exchange at the moment each order arrives. For this purpose, we use quotes disseminated by the exchanges through the Option Price Reporting Authority (OPRA). The OPRA data includes time-stamped bid-ask quotes, with depth, from all five exchanges.
Our analysis also requires intraday observations of the underlying stock prices. We compute the stock NBBO quote midpoints using quote data reported by the Securities Industry Automation Corporation (SIAC). The quotes reported by SIAC are the same as those recorded on the New York Stock Exchange’s TAQ database.

3. Fragmentation Metrics

For each order in our sample, we construct the consolidated book of all standing limit orders and dealer quotes on the opposite side of the market at the moment the order entered the system. Our reconstructed book includes all limit orders that have been entered into the system, but have not yet been reported as filled or cancelled. It includes orders submitted before the open, and “Good Until Cancelled” orders submitted on prior days. In cases where an order has received a partial execution, we continue to include the unexecuted portion of the order in our reconstructed book, until the order has been reported fully executed or cancelled. Dealer quotes are not directly observed, but are inferred from the depth of the quote disseminated by the exchange through OPRA. Specifically, we define a dealer quote to be the exchange’s advertised quoted price, with a depth equal to the exchange’s quoted depth minus the depth of all the public limit orders on that exchange at the inside price. In some cases, all the quoted depth comes from limit orders, and there is no dealer quote.

Our reconstructed books include slightly different sets of orders than the actual exchanges limit order books. For one, we include orders in our book immediately when they enter the system, while in the real world, the floor-based exchanges do not immediately route all orders to the order book. For example, if the incoming limit order is partially marketable or improves upon the prevailing quote, the trading crowd on the floor will be given an opportunity to fill the
order before it is booked. As a result, incoming orders might be displayed with a lag. On the other hand, because our data set does not include non-system orders, we do not know about some orders that would have been displayed in the real world.

Having constructed the consolidated book at the time of each incoming order, we are able to reconstruct what might have happened to incoming market orders and limit orders, had the same order been submitted to a consolidated market. We can then compare the quality of order execution in a consolidated market with the execution quality in a fragmented market, or more generally in a market with a lower degree of consolidation.

3.1 Effect of Fragmentation on Incoming Market Orders

For incoming market orders that arrive during trading hours, we determine the price at which the order would have been filled, had it interacted with the best orders and dealer quotes in the hypothetical consolidated market. That is, we determine the price at which the market order would have filled in the consolidated market, assuming the market makers offer no price improvement beyond their quoted prices. If the order actually executed at a price better than we would predict in our consolidated market, we assume that whoever filled the order would have also done so in a consolidated market at the same price. If the size of the best displayed order or dealer quote is not sufficient to fill the entire size of the incoming order, the incoming order will move on to execute the next best order, and continue to “walk up the book” until it is entirely filled. If the aggregate depth on the opposing side is insufficient to fill the incoming market order, we record the outcome as ambiguous, and make no further assumptions about what the execution price would have been.
We then repeat the process without consolidating the order book, to determine the price at which we would expect the same order to execute in a fragmented market with no price improvement. In our hypothetical reconstruction, market buy orders will execute at the same or lower price in a consolidated market, as compared to a fragmented market, while market sell orders will execute at the same or higher price. The (absolute) difference in execution price in the two markets is a measure of the potential cost imposed by fragmentation on market order investors. We would expect this measure to be a high estimate of the true cost, because it does not take into account the possibility of price improvement. Price improvement is likely in a fragmented market in cases where an order is routed to an exchange that is not at the inside quote, especially if the exchanges have “step-up” provisions in their automatic execution systems, or if exchange rules or practices censure or restrict trading outside of the best quoted price.

3.2 Effect of Fragmentation on Incoming Limit Orders

For incoming limit orders that arrive during trading hours, we follow a similar procedure, but focus on the number of contracts executed instead of on the execution price. That is, we interact the order with the consolidated book to determine whether the order could have been fully executed immediately at the limit price, or if not, how many contracts could have been executed immediately.

Again, we repeat the procedure without consolidating the book to determine how much of the order would have been executed in a fragmented market. We then compare the fill rates in the two markets. Orders that execute in the consolidated market but fail to immediately execute in a fragmented market indicate a potential cost to investors of fragmentation. The cost depends
on whether the order ultimately executes and on the delay in the execution if it does execute. We tabulate the fates of these orders.

### 3.3 Effect of Fragmentation on Standing Limit Orders

For each incoming market and limit order during trading hours, in addition to the execution statistics above, we also identify the standing limit orders on the other side of the market that would have traded against the incoming order in our hypothetical consolidated markets. We then look to see whether the same standing orders would have interacted with the incoming order in the fragmented market. If the two sets of standing orders differ, market fragmentation has led to an apparent violation of the consolidated price, public, or time priority. As a result, dealers or other investors benefit at the expense of the investors behind the disadvantaged orders. We count the number of times this happens, and we track the subsequent fate of the disadvantaged orders.

### 3.4 Order Translations

To characterize fragmentation across series within a class, we need to express all orders on a comparable basis. Using the Black Scholes model, we translated all prices and quantities into their delta equivalents in the underlying security. In particular, given assumptions about volatilities, interest rates, and dividends that we discuss below, we compute the implied stock prices that correspond to the various option prices that we analyze. With these same assumptions, we compute the delta of the option and use it translate order sizes to delta equivalent stock shares. We then analyze the translated data using the methods described above.
The translations are not particularly sensitive to the interest rate assumptions and dividend assumptions since the options contracts that we examine are all short-term contracts. Our assumptions about volatilities are more critical. As noted in the introduction, imperfections in our translation procedures can move orders closer to or further from the markets when implied volatilities depend on strike price or time to maturity. To address this problem, we model the volatility smile and the volatility term structure. Our model assumes that volatilities differ across series and days, but are constant within each series-day. Accordingly, for each series-day, we compute the time-weighted average implied volatility obtained from midpoints of the cross-market inside quotations, and use this average to produce our translations.

This simple model is subject to two significant criticisms. First, it fails to model changes in volatility that may occur within a day. Such changes will make the absolute values of the translations inaccurate. However, if the changes affect all series, they should only have a secondary effect on our analyses since the translations will preserve the relative positions of the orders. Second, our simple model fails to consider how changes in underlying prices cause series to slide up or down the volatility smile. This problem will be greatest when the underlying price change is large and the volatility smile is steep.

4. **Fragmentation across Exchanges**

In this section, we present the results of our analysis of the effects of fragmentation across exchanges. We examine the impact of fragmentation on execution quality for incoming market orders, incoming limit orders, and standing limit orders.
4.1 Incoming Market Orders

Figures 1 and 2 graphically depict our results with respect to incoming market orders. Figure 1 shows the distribution of “potential harm” to market orders, measured as the absolute difference between the per-share execution prices in our reconstructed consolidated and fragmented markets. Buy orders will execute at the same or lower price in the consolidated market, and sell orders will execute at the same or higher price. The distribution is depicted as a histogram, with damages rounded up to the nearest five cents. Thus, the orders in the box labeled zero are those that received identical executions in both markets. Those in the box labeled “.05” are orders which received inferior execution in a fragmented market, by an amount greater than zero up to and including five cents. Due to the discrete tick size, the vast majority of observations fall exactly on multiples of the tick. Those that do not are orders that walked up the book and executed at multiple prices.

Only about 40% of incoming market orders get identical execution in our reconstructed consolidated and fragmented markets. Of the 60% of orders that are potentially harmed, slightly more than half are harmed by ten cents or less. The average potential harm was XX cents. Recall that the tick size in the equity option market is five cents for option prices under $3.00 and ten cents for options above $3.00.

The measure of potential harm depicted in figure 1 is likely to overstate the true damage resulting from fragmentation, for two reasons. First, in our reconstruction, we are unable to observe hidden depth provided by market makers at or outside of the exchange’s inside quote, and so we are underestimating the amount of depth in fragmented markets. We also underestimate depth in the consolidated market, but the consequences of omitted depth will be larger for the fragmented market. Second, the measure in figure 1 assumes that market orders
get no price improvement relative to the exchange’s quote. In reality, market orders might get price improvement, especially in cases where the exchange is not at the inside quote.

To address these problems, we present an alternative measure in figure 2. This figure has the same structure as figure 1, but now we compare the execution of each order in the consolidated market with the actual execution, as reported in the audit trail. Hidden depth or price improvement in the fragmented market should be reflected in the actual execution price. In some cases, the order executes at a price better than we would predict in our consolidated market with no price improvement. In these cases, we assume that whoever filled the order would have also done so in a consolidated market. Otherwise, having no way of knowing whether there might have been price improvement in the consolidated market, we assume that there was not. As illustrated in the figure, the costs of fragmentation are considerably lower than indicated in figure 1, but still nontrivial. Now, roughly 60% of orders receive the same execution in both markets, and 40% are harmed, as compared to 40% and 60% using the previous measure.

4.2 Incoming Limit Orders

Table 2 reports our main results of our comparison of execution rates for incoming limit orders in consolidated and fragmented markets. As indicated in the table, roughly two-thirds of the orders that are marketable with respect to the consolidated book are also marketable with respect to the fragmented book. An additional 5.5% would have been partially marketable, and the remainder would not have been marketable. Measured in terms of contract volume, roughly three-quarters of the order flow that is marketable in a consolidated market would be marketable in a fragmented market. These results indicate that about one-third of incoming limit orders, and
roughly one-fourth of the contract order flow may be subject to delayed execution or non-execution as a result of fragmentation.

To measure the extent to which these orders actually experienced delayed execution or non-execution, we examine the subsequent history of these orders. Figure 3 summarizes our findings. The graph depicts how the total sample of unfilled limit orders partitions into three categories—orders that have been filled, those that have been canceled, and those that remain unfilled—as a function of time outstanding. The region between the bottom of the graph and the lowest line corresponds to executed orders. The region between the lowest line and the next line corresponds to cancelled orders, and the area above the second line corresponds to unexecuted orders.

The graph illustrates that a substantial portion of the unfilled limit order volume, slightly more than 50%, is filled within ten minutes of submission, but after that point, relatively few additional orders are executed, so that less than 58% are executed within one hour of submission. Roughly 11% of the unfilled volume is canceled within ten minutes, but the cancellation rate quickly declines thereafter. Notably, a significant portion of the unfilled volume, in excess of 28% of the original unfilled volume, remains unfilled one hour after submission.

4.3 Standing Limit Orders

Our results on the effects of fragmentation on standing limit orders are summarized in table 3. We find that a significant number of standing orders are harmed as a result of incoming orders on the other side of the market being routed to other venues. As the table indicates, some of these incoming orders would have remained unexecuted in our reconstructed version of the fragmented market (they would have locked or crossed the market). Others would have
interacted with other orders or dealer quotes, resulting in a violation of price, public, or time priority.

5. **Fragmentation across Strike Prices**

Table 4 summarizes our basic results on fragmentation across strike prices. The format of the table is comparable to table 2, but now we are comparing a market consolidated in two dimensions (exchanges and strike prices) with a market consolidated in only one dimension (strike prices).

As the table indicates, fragmentation across strike prices appears to have a significant impact on the marketability of incoming limit orders. Although not as extreme as the effects of fragmentation across exchanges, the impact is certainly worthy of note. After consolidating all order flow across exchanges, we find that roughly 18% of orders that would be executed in a consolidated strike price market would no longer be executed with fragmented strike prices. Measured in terms of trading volume, it would amount to roughly 11%.

This result has policy implications for the optimal interval between strike prices. The option exchanges have repeatedly sought to narrow the interval between strikes. The smaller the interval between strikes, the more substitute contracts are available to a trader, and greater is the potential for fragmentation.

In an effort to measure the extent to which the interval between strike prices contributes to increased dealerization, we tested for a cross-sectional relation between the dealer participation rate in our reconstructed market and the relative interval between strike prices. Specifically, we
regressed the dealer participation rate (in logistic transform) on the strike price interval, with the number of orders (in logs) as a control variable. The cross-sectional variation in the strike intervals that enables us to estimate this specification comes from three sources. First, for a fixed dollar strike-price interval, the effective interval is smaller for higher-priced stocks. Second, the strike-price interval changes from $2.50 to $5.00 as the strike prices move above $50. Third, option classes in the pilot programs described above have a narrower strike-price interval.

The results, reported in Table 5, indicate a highly significant negative coefficient on the strike interval, meaning that dealer participation rates are significantly higher when the interval between strike prices is lower. In addition, the coefficient on the control variable is negative and significant, verifying that the dealer participation rate is lower for actively-traded classes.

We recognize that a finer strike price interval provides certain benefits to investors, inasmuch as it enables them to achieve more precisely their target payoff profiles. Our results suggest that these benefits are to some degree are likely to be offset by the costs of increased dealer intermediation.

6. Conclusion

This paper explores a new approach to measuring the impact of fragmentation on order execution quality. The central idea is to take a set of orders and dealer quotes, observed in a fragmented market, and see what would have happened had the same orders and quotes been consolidated into a single market with strong price and time priority rules, and public orders trading ahead of dealers. We use this framework to measure the impact of fragmentation on the

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2 In the last few years, the exchanges have implement pilot programs that have decreased the interval between strike prices from $5.00 to $2.50 for strikes between $25 and $50, and from $2.50 to $1.00 for selected stocks trading under $20.
execution quality of both market orders and limit orders. For limit orders, we study not only the impact of fragmentation on initial marketability, but also the costs imposed on standing limit orders in the book that may face non-execution or delayed execution due to fragmentation of the incoming order flow on the other side of the market.

With respect to our specific estimates from option markets, our results indicate that prior to the implementation of the linkage plan in June 2003, there was significant room for improvement in execution quality for customer orders. Our framework provides a natural approach for evaluating the success of the linkage plan in promoting better markets and improving execution quality.

At a more fundamental level, we advocate a broader concept of fragmentation than has been typically used in prior research. The existing literature has focused primarily on fragmentation of trading in single security across different venues, or perhaps across time. More generally, fragmentation also occurs across different securities that embody similar risks.

Although this paper does not measure the benefits and costs of fragmentation, we recognize that many readers may use our results to produce their own informal—and perhaps formal—estimates. To responsibly estimate these benefits and costs, analysts must consider two sets of issues that about which this paper provides little information. First, many of the benefits of fragmentation come from the convenience that distributed markets, specialized products, and innovative trading systems provide. We provide no direct evidence on these benefits. Nor do we address the broad benefits that arise from competition across trading platforms. Second, better execution of public limit orders is a major benefit of consolidation to a price-time order precedence matching system. If markets were consolidated, traders would likely use limit orders more intensively than they presently do. Our analysis only considers limit orders as they are
presently used. Accordingly, any attempts to draw inferences about the benefits of consolidation from our results will likely underestimate those benefits.
References


### Table 1: Sample Composition

#### Panel A: Sample Census

<table>
<thead>
<tr>
<th></th>
<th>Total number of series-days</th>
<th>Daily volume</th>
<th>Open interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>Median</td>
</tr>
<tr>
<td>Total</td>
<td>596,883</td>
<td>55.96</td>
<td>0</td>
</tr>
<tr>
<td>Series with at least one system order</td>
<td>227,325</td>
<td>144.04</td>
<td>11</td>
</tr>
<tr>
<td>Series with no system orders</td>
<td>369,558</td>
<td>1.77</td>
<td>0</td>
</tr>
<tr>
<td>Current and next series</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distant series</td>
<td>98,272</td>
<td>208.29</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>94.79</td>
<td>10</td>
</tr>
<tr>
<td>Near-the-money series</td>
<td>100,327</td>
<td>241.72</td>
<td>30</td>
</tr>
<tr>
<td>Deep-in-the-money series</td>
<td>35,701</td>
<td>57.06</td>
<td>8</td>
</tr>
<tr>
<td>Deep-out-of-the money series</td>
<td>91,733</td>
<td>70.38</td>
<td>4</td>
</tr>
</tbody>
</table>

#### Panel B: Final Sample Selection

<table>
<thead>
<tr>
<th></th>
<th>Number of series-days</th>
<th>Daily volume</th>
<th>Open interest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>Median</td>
</tr>
<tr>
<td>Total</td>
<td>596,883</td>
<td>55.96</td>
<td>0</td>
</tr>
<tr>
<td>Less series with no system orders</td>
<td>1.77</td>
<td>0</td>
<td>595.29</td>
</tr>
<tr>
<td>Subtotal</td>
<td>227,325</td>
<td>144.04</td>
<td>11</td>
</tr>
<tr>
<td>Less distant series</td>
<td>129,489</td>
<td>94.79</td>
<td>10</td>
</tr>
<tr>
<td>Subtotal</td>
<td>98,272</td>
<td>208.29</td>
<td>14</td>
</tr>
<tr>
<td>Less remaining deep-in-the-money series</td>
<td>15,852</td>
<td>84.88</td>
<td>10</td>
</tr>
<tr>
<td>Less remaining deep-out-of-the-money series</td>
<td>34,476</td>
<td>59.82</td>
<td>0</td>
</tr>
<tr>
<td>Final sample</td>
<td>47,944</td>
<td>355.87</td>
<td>53</td>
</tr>
</tbody>
</table>
Table 2: Effects of Fragmentation on Marketability of Incoming Limit Orders

This table reports the number of orders that are marketable in a consolidated market and the proportion of these that would have been fully marketable, partially marketable, or not marketable in a market that is fragmented across exchanges. The statistics are reported both in terms of the number of orders and the number of contracts.

<table>
<thead>
<tr>
<th>Orders Fully Executed in Consolidated Market</th>
<th>Orders Number</th>
<th>Percent</th>
<th>Contracts Fragm.</th>
<th>Cons.</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fully Executed in Fragmented Market</td>
<td>341776</td>
<td>68.16%</td>
<td>6993295</td>
<td>6993295</td>
<td>70.68%</td>
</tr>
<tr>
<td>Partially Executed in Fragmented Market</td>
<td>26251</td>
<td>5.23%</td>
<td>602650</td>
<td>1268344</td>
<td>12.82%</td>
</tr>
<tr>
<td>Not Executed in Fragmented Market</td>
<td>133440</td>
<td>26.61%</td>
<td>0</td>
<td>1633014</td>
<td>16.50%</td>
</tr>
<tr>
<td>Total</td>
<td>501467</td>
<td></td>
<td>7595945</td>
<td>9894653</td>
<td>76.77%</td>
</tr>
</tbody>
</table>

Orders Partially Executed in Consolidated Market

<table>
<thead>
<tr>
<th></th>
<th>Orders Number</th>
<th>Percent</th>
<th>Contracts Fragm.</th>
<th>Cons.</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same size executed in Fragmented Market</td>
<td>34478</td>
<td>38.38%</td>
<td>552832</td>
<td>552832</td>
<td>28.36%</td>
</tr>
<tr>
<td>Fewer contracts executed in Fragmented Market</td>
<td>12298</td>
<td>13.69%</td>
<td>422421</td>
<td>817869</td>
<td>41.96%</td>
</tr>
<tr>
<td>Not Executed in Fragmented Market</td>
<td>43060</td>
<td>47.93%</td>
<td>0</td>
<td>578468</td>
<td>29.68%</td>
</tr>
<tr>
<td>Total</td>
<td>89836</td>
<td></td>
<td>975253</td>
<td>1949169</td>
<td>50.03%</td>
</tr>
</tbody>
</table>
Table 3: Effects of Fragmentation on Standing Limit Orders

This table summarizes the frequency of occasions where a standing limit order was disadvantaged as a result of fragmentation across exchanges. The table reports the number of standing orders that received identical execution in reconstructed and consolidated markets, and the contract volume corresponding to these orders, as well as cases where the standing order would be filled in a consolidated market but not in the fragmented market. The latter are divided into four categories, depending on what happened to the incoming order that would have interacted with the standing order in the consolidated market.

<table>
<thead>
<tr>
<th>Orders</th>
<th>Contracts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standing Orders Filled in Both Markets</td>
<td>74431</td>
</tr>
<tr>
<td>Standing Orders Harmed by Fragmentation</td>
<td></td>
</tr>
<tr>
<td>Incoming Order was not Executed in Fragmented market</td>
<td></td>
</tr>
<tr>
<td>Fully-nonexecuted orders</td>
<td>37268</td>
</tr>
<tr>
<td>Non-executed portions of partially-executed orders</td>
<td>271282</td>
</tr>
<tr>
<td>Incoming Order was Executed in Fragmented market</td>
<td></td>
</tr>
<tr>
<td>Violation of Price Priority</td>
<td>86373</td>
</tr>
<tr>
<td>Violation of Public Priority</td>
<td>24358</td>
</tr>
<tr>
<td>Violation of Time Priority</td>
<td>21611</td>
</tr>
</tbody>
</table>
Table 4: Measures of Fragmentation across Strike Prices

This table reports the frequency with which orders that are executable in a market consolidated across exchanges and strike prices are executable in a market that is consolidated across exchanges, but fragmented across strike prices. Orders were translated using an option model into delta-equivalent orders in the underlying stock, in order to make them comparable.

<table>
<thead>
<tr>
<th>Orders Fully Executed in Consolidated Market</th>
<th>Orders Equivalent Shares (100s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
</tr>
<tr>
<td>Fully Executed in Fragmented Market</td>
<td>328709</td>
</tr>
<tr>
<td>Partially Executed in Fragmented Market</td>
<td>18878</td>
</tr>
<tr>
<td>Not Executed in Fragmented Market</td>
<td>55342</td>
</tr>
<tr>
<td></td>
<td>402929</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Orders Partially Executed in Consolidated Market</th>
<th>Orders Equivalent Shares (100s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
</tr>
<tr>
<td>Same size executed in Fragmented Market</td>
<td>32811</td>
</tr>
<tr>
<td>Fewer contracts executed in Fragmented Market</td>
<td>10804</td>
</tr>
<tr>
<td>Not Executed in Fragmented Market</td>
<td>23164</td>
</tr>
<tr>
<td>Total</td>
<td>66779</td>
</tr>
</tbody>
</table>
Table 5: Relation between dealer participation rate and strike-price interval

This table presents estimates of the specification:

$$\log\left(\frac{DPR}{1 - DPR}\right) = a + b_1 \text{INTERV} + b_2 \log(\text{ORDERS})$$

where DPR is the dealer participation rate in a reconstructed market where orders are consolidated across exchanges, but not across strike prices, INTERV is the interval between strike prices, as a percentage of the stock price, and ORDERS is the total number of orders for that option class in our sample. The equation is estimated on a sample of orders for which, within our reconstruction, execution would be unaffected by strike price fragmentation. The regression $R^2$ was equal to .403.

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>INTERV</th>
<th>log(ORDERS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>3.75</td>
<td>-0.36</td>
<td>-1.97</td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(-15.10)</td>
<td>(-10.04)</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Measure of Potential Harm to Market Orders

This graph depicts the frequency of the degree to which incoming market buy and sell orders are subject to potential harm as a result of fragmentation. Potential harm is measured as the difference between execution prices in a hypothetical consolidated market and a reconstructed version of the fragmented market. In both the consolidated market and the reconstructed fragmented market, incoming orders are assumed to interact with the best available dealer quotes or limit orders, assuming no hidden depth or price improvement.
Figure 2: Measure of Apparent Harm to Market Orders

This graph depicts the frequency of the degree to which incoming market buy and sell orders appear to be harmed as a result of fragmentation. Harm is measured as the difference between execution prices in a hypothetical consolidated market and the actual execution price observed in the market. It is assumed that there is no hidden depth or price improvement in the hypothetical consolidated market, except in cases where the observed execution of the order reveals it.
Figure 3: Subsequent Disposition of Unfilled Incoming Limit Orders

This graph summarizes the subsequent execution and cancellation history for unfilled limit orders (and unfilled portions of limit orders) that would have been filled immediately in a consolidated market. The graph shows the contract volume that was subsequently filled (bottom region), canceled (middle region) and remained unexecuted (top region), as a function of time after order submission.