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Abstract

In this paper we take a portfolio approach to analyze the investment strategy of a venture capitalist (VC), and study the optimal size of a VC’s portfolio. We show that portfolio size and scope affect the incentives of both entrepreneurs to exert effort and VCs to make start-up specific investment. A small portfolio improves entrepreneurial incentives because it allows the VC to concentrate his limited human capital on a smaller number of start-ups, adding more value to each start-up. In addition, by holding a small portfolio, the VC commits not to extract higher rents from the entrepreneurs, with a positive impact on their incentives. A large and focused portfolio is beneficial for the VC because it allows the VC to reallocate his limited resources and human capital from one start-up to another in case of a start-up failure. Furthermore, a large and focused portfolio allows the VC to extract greater rents from the start-ups because the VC can induce competition for his limited resources. We show that the VC finds it optimal to limit his portfolio size when start-ups have higher quality prospects ex ante, that is, when providing strong entrepreneurial incentives is most valuable. The VC expands his portfolio size only when start-up fundamentals are more moderate and only when he can form a sufficiently focused portfolio. Finally we show that the VC may find it optimal to engage in portfolio management by divesting some of his start-ups early since this strategy allows him to extract a greater surplus from the remaining start-ups in his portfolio. Surprisingly, we find that under certain conditions, portfolio management, despite being socially inefficient ex post, improves ex ante social welfare by enlarging the set of economically viable start-ups.
1 Introduction

Existing theoretical work on venture capital (VC) has so far concentrated on a VC’s investment in a single entrepreneurial start-up, where the VC provides monetary and non-monetary resources to turn the entrepreneur’s project idea into a viable business.\(^1\) However, VC funds typically invest in more than one start-up at any given time, and engage in active portfolio management to maximize the return from their investment. In recent research, Kaplan and Schoar (2005) find substantial heterogeneity in performance across private equity funds of different size, and suggest that VCs with superior skills and greater human capital can generate better results in their investments. They also find that better performing VC funds grow proportionally slower and argue that better VCs may choose to stay small (by deliberately limiting the amount of capital raised) to avoid dilution from allocating their limited amount of human capital over a large number of start-ups.\(^2\)

In this paper we take a portfolio approach to analyze the investment strategy of a venture capitalist and investigate the optimal size of a VC’s portfolio. More specifically, we address the following questions: What determines the size of a VC’s portfolio? What are the benefits and costs of having a small versus a large portfolio? What are the strategic aspects of managing a portfolio of start-ups? Do VCs prefer having a diversified portfolio as opposed to a focused one? How do size and focus of a VC’s portfolio affect performance?

Our analysis shows that holding a small portfolio can be an optimal strategy for a VC even if the VC has access to a large number of potentially profitable start-ups in the economy. Our starting point is to recognize that VC human capital is a scarce resource in the economy which cannot be augmented easily. Even if the VC has the ability to raise unlimited amount of capital for a large number of start-ups, this may not be the optimal strategy if the VC cannot back up his monetary investment by his human capital. Spreading his human capital and diluting his value adding capability over a large number of start-ups may affect performance so adversely that the VC may find it optimal to limit the size of his portfolio.

Our analysis starts with the notion that both entrepreneurial effort and VC human capital are essential inputs for a given start-up’s success. In addition, a key feature of our analysis is to recognize that VC


\(^2\)For anecdotal evidence on the relevance of fund size see The Economist, which wrote on April 2, 2005 that “some venture capital funds say they have turned away money from investors in order to keep fund sizes down to an amount that can be managed responsibly”.

1
human capital is a fixed resource in limited supply, and cannot be expanded easily. The basic trade-off we investigate is that whether a given VC should concentrate all his human capital and resources on a small number of start-ups, or spread them over a larger number of start-ups.

We show that the size and scope of the VC’s portfolio affect both entrepreneurial incentives to exert effort and the VC’s incentives to make start-up specific investment. A small portfolio is beneficial for the VC for two different reasons. The first is that in a smaller portfolio the VC can add more value to each start-up and, as a response, each entrepreneur finds it optimal to exert higher effort, improving success potential of the start-up. The second benefit of a small portfolio is that by investing in a small number of start-ups, the VC limits his ability to induce competition among start-ups for his limited human capital and resources. In other words, by holding a small portfolio, the VC commits not to exploit the entrepreneurs by threatening to take resources away from one start-up and transferring them to another one. This commitment proves beneficial for ex-ante entrepreneurial incentives.

There are benefits associated with holding a large portfolio as well. The first is that having a larger number of start-ups increases the VC’s ex-post bargaining advantage when the start-ups compete for his limited human capital at a future project’s stage. Thus, increasing the number of start-ups in the portfolio allows the VC to extract a higher surplus from each entrepreneur. The second benefit of a large portfolio is that it allows the VC to reallocate resources from one start-up to another in case one start-up fails. We show that the magnitude of both benefits associated with a large portfolio becomes greater as the relatedness of the start-ups in the portfolio increases, that it, as the VC’s ability to form a focused portfolio increases. This follows from the fact that a more focused portfolio increases both the VC’s rent extraction ability and his resource reallocation efficiency.

Our main results hinge on the balance between the benefits and the costs of a small versus large portfolio and the VC’s ability to form a focused portfolio. We find that a small portfolio is more desirable when start-ups have a higher potential payoff, lower risk and a lower level of relatedness. These are exactly the conditions under which promoting strong entrepreneurial incentives outweighs the cost of a reduction in the VC’s rent extraction ability and resource reallocation efficiency. In contrast, when start-ups have a lower expected return, higher risk and higher degree of relatedness, it becomes more desirable for the VC to form a larger portfolio. Note that a larger portfolio weakens entrepreneurial incentives, but this proves to be less costly for start-ups with lower expected returns and higher risks, since entrepreneurial effort will be lower in such start-ups even in a small portfolio.

Our model also highlights the value of active portfolio management, a common VC practice observed
in real life. We show that a VC with a large portfolio may find it optimal to divest one of his start-ups early, even if the company’s early stage performance is positive. Early disposal of a start-up may result in the early termination of an otherwise potentially viable venture, or in its sale to another VC fund, or in an early Initial Public Offer (IPO). This portfolio management strategy is desirable from the VC’s point of view, since divesting a start-up allows the VC to add more value to the remaining start-ups in his portfolio and to extract more surplus from them. We find that the strategy of early divestiture is optimal when the start-ups in the portfolio have a high degree of relatedness and hence, when the VC has a focused portfolio. We also show that the practice of portfolio management can increase ex ante social welfare by enlarging the set of start-ups financed by the VC.

Our paper makes several novel contributions. This is the first paper, to our knowledge, which studies the interaction of size and scope of a VC’s portfolio. We analyze the costs and benefits of a large versus small portfolio as well a focused versus a diversified portfolio. Our paper shows that a VC may prefer to limit the size of his portfolio even if he has access to a large number of potentially profitable start-ups. Note that the VC’s desire to limit portfolio size is not the outcome of the assumption that the number of good projects is limited, but it derives from the benefit of providing entrepreneurs with stronger incentives. In our model the VC may prefer to limit his portfolio size precisely because expanding portfolio size will have a negative spillover effect on the existing investments.\(^3\) Furthermore, we show that the VC will find it desirable to have larger portfolios only when he can form a portfolio of sufficiently related start-ups, that is, when he can efficiently reallocate resources from one start-up to another. Since the ability to reallocate resources proves to be most valuable for start-ups with high risk and failure rates, this implies that VCs investing in high-tech and risky industries will be more likely to have larger portfolios.

Note that the contribution of our paper is not limited to VC investment only. Our paper also speaks to the more general topic of the theory of the firm by studying both project size and scope together and adds to the literature on the theory of the firm in terms of the optimal number of divisions as well as their relatedness in a given firm. Existing research on the theory of the firm and internal capital markets considers the advantages and disadvantages of firms with a large number of divisions (see, for example, Gertner, Scharfstein and Stein, 1994), but is silent about the relatedness of divisions within a firm and its impact on optimal firm size.

\(^3\)Thus our paper provides a theoretical explanation for the observation in Kaplan and Schoar (2005) that “passing up less profitable (but potentially still positive NPV projects) could only be an optimal choice for the GP if there are negative spillover effects on the inframarginal deals from engaging in these investments,” (page 1822).
In an extension of our model, we analyze how changes in the supply of VCs relative to the supply of entrepreneurial ideas affect optimal portfolio size. We show that an increase in the availability of VCs, keeping all else constant, leads to a reduction in the optimal portfolio size and an improvement in start-up success. Similarly, an increase in the supply of entrepreneurial projects in the economy results in an increase in the optimal portfolio size.

Our work is related to several papers in the recent literature. In a recent paper, Inderst, Mueller, and Muennich (2006) show that VCs may benefit from limiting the amount of capital they raise by having “shallow pockets,” since competition for limited funds provides entrepreneurs with stronger incentives, even if it allows the VC to extract more surplus. Similarly, in our paper competition between entrepreneurs for the VC’s resources (i.e., his human capital) allows the VC to extract more surplus from his start-ups. However, in our paper, in contrast to Inderst, Mueller and Muennich (2006), competition for the VC’s human capital and the VC’s ability to extract more rents affects entrepreneurs’ incentives negatively. By holding a small portfolio, the VC limits the extent of competition between start-ups and commits to extract lower rents from entrepreneurs, with a positive impact on entrepreneurial incentives. Most importantly, the main objective of our paper is to investigate the size and focus of a VC’s portfolio. Inderst, Mueller, and Muennich (2006) abstract from determining the optimal size of the VC’s portfolio by assuming a fixed number of start-ups and do not consider the benefits and costs of having a focused portfolio, a key novel feature of our model.

Our paper is also related to the work by Kanniainen and Keuschnigg (2003), further extended by Bernile and Lyandres (2003). In these papers a VC has limited resources that he can devote to his start-ups, and adding an additional start-up to the portfolio always weakens both the VC’s and the entrepreneurs’ incentives. In our model, adding a new start-up induces competition among start-ups and allows the VC to extract more surplus at the bargaining stage. However, in our paper, despite the VC’s higher rent extraction ability, a large portfolio may result in stronger incentives and be beneficial for both the entrepreneurs and the VC. This result arises due to the complementarity between entrepreneurial effort and VC’s investment incentives. In addition, in our paper, the VC’s ability to extract surplus depends on the degree of relatedness of the start-ups, and thus portfolio focus. Differently from Kanniainen and Keuschnigg (2003) and Bernile and Lyandres (2003) we derive the optimal size of a VC’s portfolio by analyzing the combined impact of portfolio size and focus on incentives.

Our work also contributes to the literature stressing the active role of VCs in adding value to their start-ups, such as Casamatta (2003), Michelacci and Suarez (2004), and Repullo and Suarez (2004), among
others. The main difference of our paper from this literature is that these papers consider the incentive problems between a single VC and a single entrepreneur while in our paper we analyze the VC’s optimal investment strategy at a portfolio level.

The paper is organized as follows. In Section 2, we describe our basic model. In Section 3.1, we examine the case where the VC has only one start-up. In Section 3.2, we study the case where the VC has two start-ups. In Section 3.3, we determine the optimal portfolio size and derive the comparative static results of the model. In Section 4, we discuss the case in which the VC engages in active portfolio management by divesting one of his start-ups early. Section 5 presents several extensions of our model and discuss the robustness of our results. Section 6 provides the empirical implications of our model. Section 7 concludes. All proofs are in the Appendix.

2 The model

We consider an economy endowed with two types of risk neutral agents: venture capitalists (VCs) and wealth-constrained entrepreneurs. Entrepreneurs are endowed with a project idea which can be turned, with the collaboration of a VC, into a final marketable product. VCs provide capital as well as other value adding activities for turning entrepreneurs’ ideas into viable businesses. We assume initially that the VC human capital is a scarce resource in the economy, and that VCs have access to a large supply of entrepreneurs with project ideas. This assumption reflects the notion that it takes time and experience to accumulate skills and human capital to become a VC. In section 5, we relax this assumption and study the impact of VC competition for entrepreneurial start-ups on optimal portfolio size.

Entrepreneurs’ project ideas can be turned into a final product in two stages. The outcome of the first stage is either a success or a failure. If the first stage is successful, then the project is developed and commercialized during its second stage. If it is a failure, it has no value and is abandoned.

There are four dates in our economy, with no discounting between the dates. At $t = 0$, the VC chooses the number $\eta$ of start-ups to invest in his portfolio. He may invest in either one or two start-ups, or he may decide to make no investment; thus, $\eta \in \{0, 1, 2\}$. The development of each start-up requires the active involvement of both the VC and the entrepreneur. At $t = 1$, the VC makes a non-contractible start-up specific investment at a personal fixed cost of $c$, with $c > 0$. The VC’s investment can be interpreted as the

\footnote{For example, on 27 November 2004, The Economist wrote: “perhaps there are simply just a few people in private equity who are very much better at it than their rivals” in explaining the substantial performance gap across private equity funds.}
effort of acquiring all the project-specific skills and human capital that add value to the start-up, including, for example, learning about the start-up’s technology and its business opportunities, and developing all the skills useful in managing the start-up. For short, we will refer to these efforts as the VC’s start-up specific investment. We assume that the VC’s initial human capital investment with one or two start-ups has the same cost $c$. This assumption captures the notion that the VC has only limited time and resources at his disposal, and that he cannot expand his investment proportionally when he has two start-ups in his portfolio rather than only one. Thus, given limited resources, the VC can either concentrate all his resources and human capital on only one start-up, or spread his resources and human capital over two start-ups, incurring the same cost $c$ in each case.

The VC’s start-up specific investment increases the value of the project, and each start-up will have a higher value with the VC’s investment than without it. For simplicity, we normalize the start-up payoff to zero if the VC does not make the initial start-up specific investment. Note that this is not a critical assumption. All we need for our results to hold is that the potential payoff from a given start-up is higher with the VC’s investment than without it. Thus, the VC’s investment and entrepreneurial effort are both necessary and complementary inputs for the success of each start-up.

Entrepreneurs play a key role during both the first and the second stage of their project. At $t = 1$, after observing the number of start-ups the VC invests in his portfolio, each entrepreneur exerts effort $p$, at a cost of $\frac{k}{2}p^2$. The parameter $k$ measures the cost of exerting effort, with $k > 1$. Entrepreneurial effort determines the success probability of the first stage of the project, which becomes known at $t = 2$.

If the first stage of a given project is a failure, the start-up is terminated and both the VC and the entrepreneur obtain zero payoffs. If the first stage is a success, the second stage needs the active participation of both the VC and the entrepreneur. If either the entrepreneur or the VC does not participate to the second stage, the project is divested. For simplicity, we normalize the project’s payoff to zero when divested. We relax this assumption in Section 5, where we allow the entrepreneur to switch to a new VC and to continue the start-up without the original VC.

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5Thus, in our setting, the start-up specific investment represents all the non-contractible VC activities that add value to a venture. For notational simplicity, we do not explicitly consider the VC’s monetary investment into the start-ups. However, our analysis could easily be modified to incorporate explicitly an initial contractible monetary outlay for each project.

6Note also that, while entrepreneurial effort $p$ is modelled as a continuous variable, with $p \in [0, 1]$, the VC’s input is a binary choice between making the initial investment, and thus paying the cost $c$, or not. We make these assumptions for simplicity, since modelling the VC’s investment as a continuous variable as well considerably reduces the analytical tractability of the model.
We assume that contracts are incomplete in that it is not possible to contract ex-ante on the participation of either the entrepreneur or the VC to the second stage of the project. This assumption implies that both the VC and the entrepreneur can withdraw \textit{at will} their involvement and human capital from the project at the second stage. Note that this assumption is plausible particularly in the context of VC investment. Neither the VC nor the entrepreneur(s) can commit ex-ante to the continuation of the project during its second stage. VCs very often finance entrepreneurial projects surrounded by great uncertainty. Not only is it very difficult to describe the final outcome of the project ex-ante, but also it is very often impossible to contract ex-ante on the level of VC’s and entrepreneurs’ involvement, the amount of human capital and resources to be allocated to the project, and the contingencies (such as the state and progress of the project) under which resources will be available to the start-ups in the future.\footnote{Thus, contracts are incomplete in the sense of Hart and Moore (1990) and Grossman and Hart (1986). We recognize that contracts are a very important aspect of venture capital financing in real life. Kaplan and Stromberg (2003) document that VCs indeed use complex contracts designed to mitigate adverse selection, moral hazard, and hold-up problems. The main assumption of our paper is that, after all these contractual features are accounted for, VC contracts contain a significant degree of residual incompleteness and therefore are subject to renegotiation.}

Entrepreneurs’ and the VC’s ability to withdraw their human capital from the project implies that it is not possible to contract on the division of the total surplus between the VC and the entrepreneurs at \( t = 0 \), and that surplus allocation is determined at the interim stage, at \( t = 2 \), through bargaining. It also implies that contracts written ex-ante between the VC and the entrepreneur(s) on how to share the final surplus, such as equity contracts (or options, as in Noldeke and Schmidt, 1998), are ineffective since both the entrepreneur and the VC can (unilaterally) withdraw their participation and human capital from the implementation phase of the project.\footnote{For further discussion of this point, see Stole and Zwiebel (1996a) and (1996b).} Even if the VC and the entrepreneur wrote, at the outset of the venture, a sharing rule on the final payoff of the start-up, the inability to contract ex-ante on the participation of the entrepreneur and the VC to the second stage of the project implies that any pre-existing sharing rule can be renegotiated away, and the division of the surplus is determined entirely by interim bargaining.

Thus, conditional on observing a successful outcome for the first stage of the project at \( t = 2 \), the VC and the entrepreneur(s) bargain over their compensation for the continuation of the start-up. For simplicity, we assume that the VC and the entrepreneur(s) have equal bargaining power.\footnote{The more general case where the VC and the entrepreneur have different bargaining power is available at request from the authors.} The outcome of the bargaining process determines the allocation of the surplus between the VC and the entrepreneur,
and thus affects their incentives.

At $t = 3$, the payoff from the project is realized and distributed between the VC and the entrepreneur(s). The payoff depends on the number of start-ups in the VC’s portfolio. If the VC invests in only one start-up and concentrates all his initial investment on one start-up only, the payoff from the start-up, if successful in the first stage and continued during its second stage, is $2\Delta$. If the VC invests in two start-ups and allocates his initial investment between the two start-ups, each start-up, if successful in the first stage and continued into its second stage, generates a payoff of $\Delta$. Note that this feature is an implication of our earlier assumption that, if the VC chooses one start-up, he can specialize all his initial investment on one start-up only, and as a result, the start-up will have a higher payoff than in the case if the VC allocates his initial investment over two start-ups. Since the cost of initial investment, $c$, is the same whether the VC invests in one or two start-ups, this assumption implies that the VC can obtain the same total potential payoff from his portfolio, which is then divided among the number of start-ups in the portfolio. This “linear” payoff structure implies that none of our results are driven by the presence of economies or diseconomies of scale in the VC’s “production technology.”

If one of the start-ups fails in its first stage, the VC can concentrate all his resources and human capital exclusively on the successful start-up, obtaining a payoff equal to $(1 + \phi)\Delta$, with $0 \leq \phi \leq 1$. The value of the parameter $\phi$ depends on the ability of the VC to transfer the start-up specific investment that he has made from one start-up to the other. Thus, $\phi$ depends on the degree of relatedness of the two start-ups, and we interpret it as representing the degree of “focus”, or scope of the VC’s portfolio.

3 Analysis

3.1 The VC invests in one start-up

Proceeding backward, we first characterize the surplus allocation between the VC and the entrepreneur through interim bargaining. When the VC invests in only one start-up, $\eta = 1$, we model the bargaining game between the entrepreneur and the VC as a standard alternating-offers game where, if bargaining breaks down, both the VC and the entrepreneur receive their outside options, which we normalize to zero.\(^\text{10}\) With equal bargaining power, the VC and the entrepreneur share equally the surplus, $2\Delta$, that they jointly generate and each obtains a payoff $\Delta$. Thus, the entrepreneur determines her level of effort $p$.

\(^{10}\text{See, for example, Binmore, Rubinstein and Wolinski (1986).}\)
by maximizing her expected profit \( \pi_E^1 \), that is

\[
\max_p \pi_E^1 = pD - \frac{k}{2} p^2. \tag{1}
\]

Correspondingly, the VC’s expected profit \( \pi_V^1 \) is

\[
\pi_V^1 = pD - c. \tag{2}
\]

**Proposition 1** If the VC makes the start-up specific investment at \( t = 1 \), the optimal level of effort exerted by the entrepreneur, \( p^1^* \), is

\[
p^1^* = \frac{D}{k}. \tag{3}
\]

The corresponding level of expected profits for the VC and the entrepreneur are

\[
\pi_V^1 = \frac{D^2}{k} - c, \quad \pi_E^1 = \frac{D^2}{2k}. \tag{4}
\]

The VC has an incentive to make the start-up specific investment only if he expects a positive expected profit. The following lemma characterizes the VC’s investment decision.

**Lemma 1** The VC makes the start-up specific investment if and only if \( D \geq D_m = \sqrt{k} \).

If the VC does not make the investment at \( t = 1 \) (that is, if \( D < D_m \)), the payoff from the project is zero, the entrepreneur does not exert any effort, and both parties obtain zero profits.

### 3.2 The VC invests in two start-ups

We proceed again backward, and first characterize the outcome of the interim bargaining game between the VC and the two entrepreneurs. When the VC invests in two start-ups, \( \eta = 2 \), the bargaining process between the VC and the entrepreneurs depends on whether only one or both projects have a successful outcome at their first stage, \( t = 2 \). There are three different possible cases (states of the world): (i) both projects are successful in their first stage, state \( SS \), (ii) one project is successful while the other one is a failure, state \( SF \), (iii) both projects are a failure, state \( FF \).

We begin our analysis with the (simpler) case where only one start-up, say start-up \( i \), is successful, state \( SF \). In this case, the VC can reallocate his human capital and concentrate exclusively on start-up \( i \), increasing its payoff from \( D \) to \( (1 + \phi) D \). Since the VC has only one successful start-up in his portfolio,
the entrepreneur and the VC engage in bargaining with alternating offers (with no outside options) as in the previous section. Thus, the VC’s and the entrepreneurs’ payoffs, denoted respectively by $l_{V_i}^2(SF)$, $l_{E_i}^2(SF)$ and $l_{E_j}^2(SF)$, are

$$l_{V_i}^2(SF) = \frac{(1 + \phi) \Delta}{2}, \quad l_{E_i}^2(SF) = \frac{(1 + \phi) \Delta}{2}, \quad l_{E_j}^2(SF) = 0. \quad (5)$$

If both start-ups are successful at $t = 2$, state $SS$, the VC bargains with both entrepreneurs. We model this process of “multilateral” bargaining between the VC and the two entrepreneurs as in Stole and Zwiebel (1996a). We assume that the VC leads individual bargaining sessions with one start-up at a time, starting, say, with entrepreneur $i$. Individual bargaining occurs again as an alternating offer game. If the VC and entrepreneur $i$ reach an agreement, the VC starts a round of bargaining with entrepreneur $j$. If bargaining between the VC and entrepreneur $i$ breaks down without an agreement, entrepreneur $i$ drops from the bargaining process and the VC engages in bargaining with entrepreneur $j$, where both players have zero outside options. Equilibrium payoffs are subject to the (stability) condition that if the VC reaches an agreement with entrepreneur $i$, and bargaining with entrepreneur $j$ breaks down, then entrepreneur $j$ drops from the bargaining process and the VC and entrepreneur $i$ renegotiate their original agreement through bargaining, where now both players have zero outside options. This condition ensures that the agreement reached between the VC and entrepreneur $i$ (resp. $j$) anticipates the renegotiation that would take place if bargaining between entrepreneur $j$ (resp. $i$) and the VC breaks down. The VC’s and the entrepreneurs’ payoffs, denoted respectively by $l_{V_i}^2(SS) \equiv l_{V_i}^2(SS) + l_{V_j}^2(SS)$, $l_{E_i}^2(SS) \equiv \Delta - l_{V_i}^2(SS)$, $i = 1, 2$, are characterized by (see Stole and Zwiebel, 1996a)

$$\Delta - l_{V_i}^2(SS) = l_{V_j}^2(SS) - \bar{l}_V, \quad \Delta - l_{V_j}^2(SS) = l_{V_i}^2(SS) - \bar{l}_V,$$

where

$$\bar{l}_V = \frac{(1 + \phi) \Delta}{2} \quad (6)$$

represents the VC’s outside option when bargaining with entrepreneur $j$. Solving the above system of equations, we obtain

$$l_{V_i}^2(SS) = \frac{(3 + \phi) 2 \Delta}{6}, \quad l_{E_i}^2(SS) = \frac{(3 - \phi) \Delta}{6}, \quad i = 1, 2. \quad (7)$$

By examining the VC’s payoff in the $SS$ state (7), it is easy to see that when the VC has two successful start-ups in his portfolio he obtains a greater fraction of the total surplus than when he has only one
successful start-up, that is, \( \frac{3+\phi}{6} \geq \frac{1}{2} \) for all \( 0 \leq \phi \leq 1 \). This happens because having a second successful start-up in his portfolio gives the VC an outside option while bargaining with each entrepreneur. Hence, the presence of a second start-up and the ability to transfer resources from one start-up to the other creates “competition” between entrepreneurs, allowing the VC to extract more surplus. The VC’s ability to transfer ex-post resources from one start-up to another is critical, since when \( \phi = 0 \) the VC extracts the same fraction of the surplus with both one and two start-ups. Note also that the VC’s surplus, \( I_v^2(SS) \), is increasing in the degree portfolio focus, \( \phi \) whereas each entrepreneur’s surplus, \( I_{E_i}^2(SS) \), is decreasing in the level of portfolio focus, \( \phi \). Thus, a greater level of portfolio focus \( \phi \) benefits the VC but hurts the entrepreneurs at the bargaining stage. This happens because a greater level of focus leads to a greater outside option for the VC while he bargains with each entrepreneur, allowing the VC to extract a greater fraction of the total surplus. Note however that, as we will show below, the entrepreneurs will benefit ex-ante from a greater degree of focus.

Finally, if both entrepreneurs fail in the first stage, state \( FF \), both start-ups are terminated and all agents obtain zero payoffs.

We now characterize the entrepreneurs’ choice of effort. If the VC makes the specific investment for each start-up, anticipating her payoffs in different states of the world, entrepreneur \( i \) determines her effort level by maximizing her expected profit \( \pi_{E_i}^2 \), that is

\[
\max_{p_i} \pi_{E_i}^2 \equiv p_ip_j \left( 3 - \phi \right) \frac{\Delta}{6} + p_i(1-p_j) \left( 1 + \phi \right) \frac{\Delta}{2} - \frac{k}{2} p_i^2; \quad i, j = 1, 2; i \neq j. \tag{8}
\]

Similarly, the VC’s expected profit \( \pi_v^2 \) is

\[
\pi_v^2 \equiv p_ip_j \left( 3 + \phi \right) \frac{2\Delta}{6} + p_i(1-p_j) \left( 1 + \phi \right) \frac{\Delta}{2} + p_j(1-p_i) \left( 1 + \phi \right) \frac{\Delta}{2} - c; \quad i, j = 1, 2; i \neq j. \tag{9}
\]

The first-order condition of \( 8 \) is

\[
p_i(p_j) = \frac{(3 + \phi) - 4\phi p_j \Delta}{6k}. \tag{10}
\]

Note that the effort exerted by entrepreneur \( i \) decreases in the effort exerted by entrepreneur \( j \) and hence, the effort levels are strategic substitutes. This happens because, when the VC has two start-ups in his portfolio, in state \( SS \) he extracts a higher surplus from each entrepreneur, reducing their expected profits and their incentives to exert effort.

\footnote{Note also that the payoffs in (7) are the same as the players’ Shapley value of the corresponding cooperative game.}
Proposition 2 If the VC makes start-up specific investment for each start-up at \( t = 1 \), the Nash-equilibrium level of effort, denoted by \( p^{2*} \), is

\[
p^{2*} = \frac{3(1 + \phi)\Delta}{2(2\phi\Delta + 3k)}.
\]

The corresponding level of expected profits for the VC and the entrepreneurs are

\[
\pi^2_{V} = \frac{3(\phi\Delta + 3k)(1 + \phi)^2\Delta^2}{2(2\phi\Delta + 3k)^2} - c,
\]

\[
\pi^2_{E_1} = \pi^2_{E_2} = \left[\frac{3((1 + \phi)\Delta)}{2(2\phi\Delta + 3k)}\right]^2 \frac{k}{2}.
\]

It is easy to verify that the equilibrium level of entrepreneurial effort \( p^{2*} \), is increasing in the degree of portfolio focus, \( \phi \). The focus parameter \( \phi \) has two opposing effects on the level of effort chosen by the entrepreneur. On the one hand, a higher degree of focus allows the VC to extract more surplus from each entrepreneur in the SS state, with a negative effect on entrepreneurial effort. On the other hand, a more focused portfolio allows the VC to reallocate more efficiently his resources to the successful start-up in the SF state, where only one of the start-ups is successful in its first stage, with a positive effect on entrepreneurial effort. As it turns out, the second effect dominates the first effect and the overall impact of an increase in focus on the level of effort and the expected profits is always positive.

Entrepreneurial incentives to exert effort and, in turn, the VC’s investment incentives depend on the number of start-ups in the VC’s portfolio. An important question is whether the entrepreneurs have stronger incentives to exert effort when the VC has one or two start-ups in his portfolio.

Lemma 2 If the VC is induced to make the necessary start-up specific investment with both one and two start-ups in his portfolio, each entrepreneur always has greater incentives to exert effort when his start-up is the only start-up in the VC’s portfolio:

\[
p^{1*} > p^{2*}.
\]

Furthermore, the difference between the levels of effort, \( p^{1*} - p^{2*} \), increases in project payoff, \( \Delta \), and decreases in the degree of focus, \( \phi \), and in the entrepreneur’s cost of exerting effort, \( k \).

If the VC makes the start-up specific investment, entrepreneurial incentives to exert effort are always lower when the VC has two start-ups rather than when he has only one. This is due to the fact that, with two start-ups in his portfolio, the VC adds less value to each start-up and is able to extract more surplus from the entrepreneurs. Thus, conditional on the VC making the start-up specific investments,
entrepreneurial incentives are always worse when the start-ups belong to a large portfolio, leading to a lower level of effort.\footnote{Note that each entrepreneur has an incentive to exert effort only if she expects the VC to make the start-up specific investment. In turn, the VC is willing to make the start-up specific investments only if he expects a positive profit, net of his total investment cost $c$.}

The difference between the level of effort in the two cases is increasing in the project payoff, $\Delta$. This property is due to the fact that when the VC has two start-ups each entrepreneur benefits less from an increase in the project payoff since the VC can extract a greater fraction of the incremental surplus from the entrepreneurs. The difference between the level of effort in a small and large portfolio is decreasing in the level of focus, $\phi$. This can be seen by noting that an increase in the degree of focus, $\phi$, increases $p^{2*}$ while it has no effect on $p^{1*}$, reducing the difference between the two effort levels. Finally, an increase in the cost of effort, $k$, always reduces entrepreneurial effort, but relatively more when the VC has only one start-up.

It is important to note that, due to the complementarity between entrepreneurial effort and the VC’s investment, it is possible that each entrepreneur exerts greater effort when the VC has a large portfolio rather than a small one, despite the fact that they obtain lower rents in a larger portfolio. This is because, under some conditions, the VC has incentives to make the start-up specific investment only when he holds a large portfolio, leading the entrepreneurs to exert effort as well. More specifically, this possibility arises when the VC’s expected profit from investing in a single start-up is negative, while his expected profit from investing in two start-ups is positive. In this case, the VC can recover his initial cost $c$ only if he holds a large portfolio. Hence, the VC will be willing to incur the cost of his initial investment and the entrepreneurs will exert effort only if the VC invests in two start-ups. We will elaborate on this possibility in more detail in the next section.

\subsection*{3.3 Optimal portfolio size}

The VC chooses his portfolio size as a result of the interaction of three distinct effects and their impact on incentives. The first one is the rent extraction effect: the VC can extract greater rents when he has a larger portfolio. This effect always induces the VC to prefer (all else equal) a larger portfolio. The second effect is the resource allocation effect: by investing in two, rather than only one start-up, the VC can reallocate ex-post his resources and human capital from one start-up to the other. The strength of this effect depends on the degree of focus of the portfolio, $\phi$. If the success probability of each start-up is fixed and the same
regardless of whether the VC has one or two start-ups, this effect always leads the VC to prefer a large portfolio to a small one.\footnote{This property can be seen as follows. If the VC has only one start-up, and the entrepreneur exerts effort $p$, total expected value of the VC’s portfolio is $p2\Delta$. If the VC has two start-ups in his portfolio, and each entrepreneur exerts effort $q$, the total expected value of the VC’s portfolio is $2q^2\Delta + 2q(1-q)(1+\phi)\Delta = 2q\Delta + 2q(1-q)\phi \Delta$. It is easy to see that the expected value of the VC’s portfolio is larger with two start-ups than with only one when $\phi > \frac{p-q}{q(1-q)}$, which is always the case when $p = q$ and $\phi > 0$. Note, however, that in our analysis, $p$ and $q$ are determined endogenously as a function of portfolio size and focus.} The third effect is the value dilution effect: a larger portfolio requires the VC to spread his fixed amount of resources and human capital over a larger number of start-ups. The result is that the VC adds lower value to each start-up: he will add only $\Delta$ to each start-up in the $SS$ state, and $(1 + \phi)\Delta$ in the $SF$ state. Thus, the value dilution effect favors a small portfolio.

Portfolio size affects the VC’s and the entrepreneurs’ incentives as follows. First, in a large portfolio, the rent extraction effect favors the VC and, thus, impacts entrepreneurial incentives negatively and the VC’s investment incentives positively. Second, a large portfolio allows the VC to reallocate his human capital from one start-up to another in case one of the start-ups fails; this possibility benefits both the entrepreneurs and the VC and, thus, affects their incentives positively. Furthermore, this effect is stronger when the level of the portfolio focus is higher. Third, in a large portfolio, dilution from spreading the VC’s resources and human capital over two start-ups lowers the payoff from exerting effort, and thus reduces both entrepreneurial effort and the VC’s investment incentives.

The VC’s optimal portfolio size depends on the value of the project payoff, $\Delta$, and portfolio focus, $\phi$, which may fall in one of three possible regions (see Figure 1): the VC can invest in no start-up at all (Region 0), in one start-up (Region 1), or in two start-ups (Region 2), as summarized in the following proposition.

**Proposition 3** There are critical values $\{\phi_c, \Delta_1(\phi, k), \Delta_2(\phi, k)\}$ (defined in the appendix) such that the VC’s optimal portfolio size is as follows:

i) for low project payoff ($0 \leq \Delta < \Delta_1$) the VC invests in no start-up, $\eta^* = 0$ (Region 0);

ii) for high project payoff ($\Delta \geq \Delta_2$) the VC invests in one start-up only, $\eta^* = 1$ (Region 1);

iii) for moderate project payoff and high focus ($\Delta_1 \leq \Delta < \Delta_2$ and $\phi_c \leq \phi < 1$) the VC invests in two start-ups, $\eta^* = 2$ (Region 2).

Furthermore, $\frac{\partial \Delta_1(\phi, k)}{\partial \phi} \leq 0$ , $\frac{\partial \Delta_2(\phi, k)}{\partial \phi} \geq 0$, and $\frac{\partial \Delta_2(\phi, k)}{\partial k} \geq 0$. 
Two key insights emerge from Proposition 3. The first is that the VC finds it optimal to hold a small portfolio when the project payoff, \( \Delta \), is relatively high, that is, when \( \Delta \geq \Delta_2 \) (Region 1). In this region, the benefits of a small portfolio in terms of better entrepreneurial incentives dominate the advantages of a large portfolio in terms of rent extraction and resource reallocation. The intuition for this result is as follows. From Lemma 2, we know that the difference in the entrepreneurs’ effort levels in a small and a large portfolio, that is, \( p^{1*} - p^{2*} \), is greater when project payoff \( \Delta \) is larger. This implies that the negative impact on entrepreneurial incentives of holding a large portfolio is greater at higher values of \( \Delta \). The proposition shows that, for start-ups with a large \( \Delta \), the VC prefers to give up the greater rent extraction ability and the resource allocation advantages of a large portfolio for the benefits of stronger entrepreneurial incentives of a small portfolio.

Note that the parameter \( \Delta \) can be interpreted as representing the project’s residual expected value, after the first stage is completed. Thus, a greater value of \( \Delta \) characterizes start-ups with either larger ultimate payoff, or with greater ultimate success probability. In other words, greater values of \( \Delta \) represent start-ups with stronger ex-ante fundamentals. Proposition 3, therefore, implies that the VC finds it optimal to have a smaller portfolio when he has access to start-ups with strong fundamentals. By holding a small portfolio, the VC boosts entrepreneurial incentives and increases the success probability of his start-ups. A small size portfolio, therefore, is desirable precisely because it allows the VC to obtain superior ex-post performance from his investment.

The second insight of Proposition 3 is that the VC finds it optimal to hold a large portfolio when the project payoff \( \Delta \) is moderate and when he can form a portfolio with sufficient focus, that is, when \( \Delta_1 \leq \Delta < \Delta_2 \) and \( \phi_c \leq \phi \leq 1 \) (Region 2). For start-ups with a more moderate potential, the benefits of a larger portfolio in terms of greater rent extraction and resource reallocation ability dominate the incentive advantage of a small portfolio. The intuition for this result is as follows. In this region, a large value of \( \phi \) implies that the rent extraction and resource reallocation effects of a large portfolio are significant. Furthermore, the difference between the levels of entrepreneurial effort in a small and a large portfolio is smaller at moderate values of \( \Delta \), as established in Lemma 2. Thus, when parameter values fall in this region, the rent extraction and resource allocation effects dominate the incentive effect, and the VC holds a large portfolio. This also implies that, for a given project payoff \( \Delta \), the VC finds it optimal to expand his portfolio only at sufficiently high values of \( \phi \) (see again Figure 1). Thus, the VC is willing to increase the size of his portfolio only if he can form a portfolio with sufficient focus by investing in highly related start-ups, a property which is formally reflected by the fact that \( \frac{\partial \Delta_2(\phi,k)}{\partial \phi} \geq 0 \). In addition, \( \frac{\partial \Delta_2(\phi,k)}{\partial k} \geq 0 \) implies
that an increase in the entrepreneurs’ cost of exerting effort, \( k \), makes larger portfolios more desirable. This happens because an increase in the value of \( k \) reduces entrepreneurial effort, which leads to a lower success probability for each start-up and to a riskier portfolio. As a result, the VC’s willingness to hold a larger portfolio increases since a lower success rate for each start-up increases the importance of the resource reallocation benefit of large portfolios. This also implies VCs prefer larger and more focused portfolios for start-ups with moderate fundamentals.

Note also that for some parameter values in this region, that is, when \( \Delta_1 \leq \Delta < \Delta_m \), the VC’s expected profits are positive only if he holds a large portfolio, and negative if he holds a small portfolio (that is, \( \pi_{1}^{-} < 0 \) and \( \pi_{2}^{+} > 0 \)). This happens because, with a small portfolio, the VC cannot extract enough rents from the entrepreneur to compensate him for the cost of making the initial investment, \( c \). In this case, anticipating that the VC is not willing to make the initial start-up investment, the entrepreneur does not exert effort either, and the project is not undertaken even if it is potentially profitable. Investing in a large portfolio, however, provides the VC with the rent extraction and resource reallocation benefits and induces him to make the required initial investment. Anticipating the improved incentives of the VC, the entrepreneurs exert effort and the projects become viable. As a result, both the VC and the entrepreneurs turn out to be better off when the VC holds a large portfolio.

The above result has an interesting implication that entrepreneurial ideas with a moderate value may be economically viable only if the VC can form a large portfolio with a sufficient degree of focus. If we interpret \( \Delta \) as measuring the size of a start-up, this implies that VCs would be willing to invest in small businesses only if they are able to combine such start-ups in a portfolio of sufficient size and focus. It also implies that entrepreneurs with smaller businesses will have an incentive to cluster in similar or related industries so that they can be financed by a common VC. Thus, small and risky businesses (characterized by small or moderate \( \Delta \)) may be economically viable and obtain VC financing only if they have a sufficient degree of industry focus among them. A policy implication from this result is that encouraging small business creation in the same or related industries will improve available VC financing and enhance social welfare because potential VCs will be willing to provide monetary and human capital to such businesses only if they have a common industry focus.

Finally, when the level of \( \Delta \) is very low, that is when \( 0 \leq \Delta < \Delta_1 \) (Region 0), the VC does not invest in any start-up. In this region the project payoff is so low that the VC cannot recover his initial investment cost \( c \). As a result, the VC does not make any start-up specific investment and the entrepreneurs do not exert any effort. Hence the project opportunities cannot be exploited.
4 Portfolio Management

In this section we show that the VC can increase his expected profits by engaging in active portfolio management, that is, by divesting one of his successful start-ups. This strategy can be optimal since the possibility of divesting one of the start-ups allows the VC to extract more surplus from the remaining one. Thus, this section helps shed some light on why VCs may make seemingly socially inefficient decisions by terminating some of their start-ups prematurely in order to maximize their own welfare.\footnote{See, for example, the Economist, November 27, 2004 which reports that “Google’s founders would have preferred to wait longer to do their IPO, but had to rush it because venture capitalists, including Kleiner Perkins, wanted to cash in.”}

We modify our basic model as follows. Consider the case in which the VC invests in two start-ups and both entrepreneurs have a successful first stage, state $SS$. The VC now faces two choices. He can either continue both start-ups, or divest one of them and dedicate himself entirely to the remaining one. If the VC chooses not to continue a successful start-up, he can divest it, for example through a sale to another VC (or some private buyer), or even take it public in an IPO. We assume that the proceeds from divesting the start-up are lower than the proceeds from continuing with the original VC and, for simplicity, are normalized to zero.\footnote{Normalizing divestiture payoff is only a simplification. Our results go through as long as divestiture payoff is lower than the payoff possible with the incumbent VC. This assumption reflects that the incumbent VC, because of the initial specific investment he made, can generate a greater payoff than the divestiture payoff.} The VC now has the option to bargain with one entrepreneur for the continuation of only her start-up and the termination of the other start-up, which we denote as “bilateral” bargaining. Alternatively, the VC can engage, as before, in multilateral bargaining with both entrepreneurs for the continuation of both start-ups. This choice is important because the VC may be able to extract a different surplus depending on whether he continues one or both start-ups.

We model the process of bilateral bargaining in state $SS$ as follows. The VC selects, with equal probability, one of the two successful start-ups, say start-up $i$, and negotiates with entrepreneur $i$ the payoff that he will receive for his exclusive participation to the continuation of start-up $i$ only. This may be achieved, for example, by negotiating, at the bargaining stage, an agreement between the VC and the entrepreneur that limits the VC’s ability to participate in other start-ups. This implies that the VC can commit not to participate to the continuation of the other project and to divest it. The VC’s ability to make such a commitment at this stage of the game (after the realization of the state of the world) is a much weaker requirement than the assumption that the VC can, at the beginning of the game, commit to continue a start-up under predetermined circumstances, a possibility that we have ruled out.\footnote{Note that the use of such provisions is very common in stock purchase agreements. For a discussion of covenants in stock purchase agreements, see, for example, the book by Fishbein and Oum.} While...
bargaining with entrepreneur $i$, the VC has the outside option to go to the start-up $j$ and start a new round of bargaining with entrepreneur $j$, obtaining $\bar{R}_j^V$, defined in (6). Both entrepreneurs have again zero outside options. This implies that the VC’s and entrepreneur $i$’s payoffs are given by

$$l_i^2B(SS) \equiv \bar{R}_V + \frac{1}{2} \left[ (1 + \phi)\Delta - \bar{R}_V \right] = \frac{3(1 + \phi)\Delta}{4},$$

(14)

$$l_j^2B(SS) \equiv (1 + \phi)\Delta - l_j^2B(SS) = \frac{(1 + \phi)\Delta}{4}, \quad l_j^2B(SS) = 0.$$  

(15)

A critical question is whether the VC can obtain a greater payoff by continuing both start-ups or by divesting one of them and continuing only the remaining one. This choice depends on whether the VC can extract more surplus by engaging in multilateral or bilateral bargaining with the entrepreneurs, given the structure of the bargaining games.\(^1\)

**Proposition 4** The VC continues only one project and divests the other if and only if $\phi \geq \frac{2}{3}$.\(^1\)

When the VC divests one of the start-ups, from (14), he will receive $\frac{3}{4}$ of the total surplus $(1 + \phi)\Delta$. When the VC continues both start-ups, from (7), he will receive a fraction $\frac{3 + \phi}{6}$ of the total surplus $2\Delta$. By direct comparison, it is easy to see that $\frac{3}{4} > \frac{3 + \phi}{6}$ for all $0 \leq \phi \leq 1$. This implies that, divesting one of the start-ups increases the VC’s rent extraction ability, but at the cost of reducing total surplus from $2\Delta$ to $(1 + \phi)\Delta$. Proposition 4 states that when the loss from divesting one of the start-ups is not too large, that is, when $\phi \geq \frac{2}{3}$, the VC finds it optimal to exploit the better bargaining position provided by bilateral bargaining, and thus prefers to continue one start-up only. When there is no value loss in reallocating resources from one start-up to the other, that is, when $\phi = 1$, the VC always prefers to divest one of the two start-ups. In contrast, when the loss from divesting one project is sufficiently large, $\phi < \frac{2}{3}$, the VC prefers to give up the better bargaining position provided by bilateral bargaining in order to realize the full potential of his portfolio and continues both start-ups.

If $\phi < \frac{2}{3}$, the VC will choose in the $SS$ state to continue both start-ups, and Proposition 3 will remain valid. Thus, in the remainder of the analysis, we will assume that $\phi \geq \frac{2}{3}$ and characterize the optimal portfolio size when the VC divests one of the successful start-ups in the $SS$ state. In the $SF$ and $FF$ states, the game unfolds as before.

\(^1\)In other words, given the structure of the respective bargaining games, the VC may be willing to inefficiently terminate one of the two start-ups if he cannot internalize a sufficiently large part of the efficiency gains that can be obtained by continuing both start-ups. The question of whether the VC can internalize, through multilateral bargaining, a sufficient portion of the efficiency gains to induce him to always make the socially efficient decision is ultimately an empirical one.

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\(^1\)Shareholders agreements, see Gompers and Lerner (1996) and Chemla, Habib and Ljungqvist (2006).
Anticipating her payoffs in different states of the world, entrepreneur $i$ determines her level of effort $p_i$ by maximizing her expected profit $\pi_{E_i}^{2B}$, that is

$$\max_{p_i} \pi_{E_i}^{2B} = p_i p_j \frac{(1 + \phi) \Delta}{4} + p_i (1 - p_j) \frac{(1 + \phi) \Delta}{2} - \frac{k}{2} p_i^2; \quad i, j = 1, 2; i \neq j. \quad (16)$$

Correspondingly, the VC’s expected profit $\pi_{V}^{2B}$ is

$$\pi_{V}^{2B} = p_i p_j \frac{3(1 + \phi) \Delta}{4} + p_i (1 - p_j) \frac{(1 + \phi) \Delta}{2} + p_j (1 - p_i) \frac{(1 + \phi) \Delta}{2} - c; \quad i, j = 1, 2; i \neq j. \quad (17)$$

**Proposition 5** The Nash-equilibrium level of effort, denoted by $p_2^{2B}$, is

$$p_2^{2B} = \frac{4(1 + \phi) \Delta}{3(1 + \phi) \Delta + 8k}. \quad (18)$$

The corresponding level of expected profits for the VC and the entrepreneurs are

$$\pi_{V}^{2B*} = \frac{8((1 + \phi) \Delta + 4k)(1 + \phi)^2 \Delta^2}{(3(1 + \phi) \Delta + 8k)^2} - c; \quad (19)$$

$$\pi_{E_1}^{2B*} = \pi_{E_2}^{2B*} = \frac{8k(1 + \phi)^2 \Delta^2}{(3(1 + \phi) \Delta + 8k)^2}. \quad (20)$$

The following proposition characterizes the VC’s optimal portfolio size.

**Proposition 6** Let $\frac{2}{3} \leq \phi \leq 1$ and $c < \bar{c}(k)$ (defined in the appendix). There are critical values $\{\Delta_1^B(\phi, k), \Delta_2^B(\phi, k)\}$ (defined in the appendix) such that the VC’s optimal portfolio size is as follows:

i) for low project payoff ($0 \leq \Delta < \Delta_1^B$) the VC does not invest in any start-up, $\eta^* = 0$;

ii) for high project payoff ($\Delta \geq \Delta_2^B$) the VC invests in one start-up only, $\eta^* = 1$;

iii) for moderate project payoff and high focus ($\Delta_1^B \leq \Delta < \Delta_2^B$ and $\frac{3}{5} \leq \phi \leq 1$) the VC invests in two start-ups, $\eta^* = 2$.

Furthermore, $\frac{\partial \Delta_1^B(\phi, k)}{\partial \phi} \leq 0$ and $\frac{\partial \Delta_2^B(\phi, k)}{\partial \phi} > 0$.

Proposition 6 mirrors Proposition 3, with one difference that when both start-ups are successful (state SS) the VC will divest one of them. The following proposition compares Proposition 3 and Proposition 6 and illustrates the effect of portfolio management on the VC’s portfolio size.

**Proposition 7** Let $\frac{3}{5} \leq \phi \leq 1$. There is a $\hat{\phi}$ such that:

i) if $\hat{\phi} < \phi \leq 1 : \Delta_1^B < \Delta_1$ and $\Delta_2^B > \Delta_2$;

ii) if $\frac{3}{5} \leq \phi \leq \hat{\phi} : \Delta_1^B \leq \Delta_1$ and $\Delta_2^B \leq \Delta_2$. 

19
When start-ups are highly related, that is, when $\hat{\phi} < \phi \leq 1$, the region in which the VC invests in two start-ups is larger with portfolio management, implied by $\Delta^B_2 > \Delta_2$. This happens because, when $\phi$ is large the VC can form a focused portfolio and can divest one of the successful start-ups with little efficiency loss. Thus, active portfolio management, by enhancing the VC’s ex-post rent extraction ability, induces him to choose a larger portfolio ex-ante. Note that the greater rent extraction ability due to active portfolio management also means that the VC has stronger incentives to make the initial start-up specific investment at $t = 1$. Thus, active portfolio management enlarges the set of feasible start-ups, implied by $\Delta^B_1 < \Delta_1$. This means that there are start-ups with a low payoff, i.e., those with $\Delta \in (\Delta^B_1, \Delta_1)$, that are economically viable only when the VC can increase his rent extraction ability by engaging in portfolio management.\textsuperscript{19} Hence, even though portfolio management results ex-post in a socially inefficient termination of a start-up, it may nevertheless improve ex-ante social welfare by enlarging the set of feasible start-ups.

The above result is reversed when start-ups are only moderately related, that is, when $\frac{3}{5} \leq \phi \leq \hat{\phi}$. In this case portfolio management shrinks the region where the VC invests in two start-ups, implied by $\Delta^B_2 \leq \Delta_2$. Divesting a successful start-up is now more costly, because the VC cannot reallocate his resources efficiently from one start-up to the other. Since the VC anticipates the cost of inefficient termination of start-ups, he chooses a smaller portfolio ex-ante. Correspondingly, for this range of parameters, the possibility of active portfolio management shrinks the set of feasible start-ups compared to the case where the VC cannot engage in portfolio management, implied by $\Delta^B_1 \geq \Delta_1$.

\section{Extensions and robustness}

In our basic model we study the static portfolio problem of a single VC facing a large supply of entrepreneurial projects. In this section we extend our model in two directions: First, we introduce VC competition for entrepreneurs and examine its effect on optimal portfolio size. Competition among VCs for start-ups generates an outside option for entrepreneurs and, thus, affects both entrepreneurial incentives and the optimal portfolio size. Second, we consider a dynamic, infinite-horizon version of our basic model, where the VC can invest in new start-ups over time when a project either fails or is completed. We show that the main results of our static model carry over to these more general settings. For analytical tractability

\textsuperscript{19}Note that, in this case, the entrepreneurs as well are ex-ante better-off. This is because the VC will be willing to finance their start-ups only if he can engage in portfolio management, even if this implies that one of the start-ups is divested ex post in the $SS$ state.
we analyze these two cases separately and, for notational simplicity, we set \( k = 1 \).

### 5.1 Competition for entrepreneurs

This section extends our basic model such that VCs compete for entrepreneurs first ex-ante, when they form their portfolios at \( t = 0 \), and then a second time in the interim, when the first stage of the project is completed. We modify our model as follows. First, we assume that a successful entrepreneur can, at \( t = 2 \), turn to a new VC and still continue her start-up in the second stage. The ability to complete the project with an alternative VC gives the entrepreneur an outside option while bargaining with the original VC. Switching to a new VC, however, results in a less valuable start-up than staying with the original VC that made the initial start-up specific investment. This assumption reflects the notion that, because of the relationship-specific nature of the human capital investment made at \( t = 1 \), the surplus created from switching to a new VC is strictly smaller than the surplus obtained by continuing with the original VC. This implies that the entrepreneur and the original VC will find it in their best interest to reach an agreement at the interim bargaining stage despite the entrepreneur’s ability to continue with a new VC. Formally, we assume that if an entrepreneur switches to a new VC, she obtains a payoff of \( \theta_1 2 \Delta \) (with \( 0 < \theta_1 < 1 \)) in the case where the original VC has one start-up, and \( \theta_1 \Delta \) in the case where the original VC has two start-ups in his portfolio. The VC has the same outside options as in the basic game.

If at \( t = 0 \) the VC invests in only one start-up, at the interim bargaining stage the entrepreneur obtains \( \bar{p}_E = (1 + \theta_1) \Delta \) due to her ability to switch to a new VC and the VC obtains \( \bar{p}_V = (1 - \theta_1) \Delta \). The entrepreneur chooses her level of effort \( p \) to maximize her expected profit \( \bar{\pi}_E^1 \), that is

\[
\max_p \bar{\pi}_E^1 = p(1 + \theta_1) \Delta - \frac{p^2}{2},
\]

resulting in \( \bar{p}_E^1 = (1 + \theta_1) \Delta \). The corresponding level of expected profits for the VC and the entrepreneur are

\[
\bar{\pi}_V^1 = (1 - \theta_1^2) \Delta^2 - c; \quad \bar{\pi}_E^1 = \frac{(1 + \theta_1)^2 \Delta^2}{2}.
\]

If at \( t = 0 \) the VC invests in two start-ups, he will bargain again with the two entrepreneurs, each having the option to leave the original VC and switch to another VC. By modifying the multilateral bargaining game discussed in the basic model correspondingly, it is easy to show that each entrepreneur’s payoff from bargaining in the SS state, denoted by \( \bar{p}_E^c(SS) \) and in the SF, denoted by \( \bar{p}_E^c(SF) \) state, are

\[
\bar{p}_E^c(SS) = \frac{\Delta}{2} + \frac{\Delta (\theta_1 - \phi)}{6}; \quad \bar{p}_E^c(SF) = \frac{\Delta}{2} + \frac{\Delta (2\phi + \theta_1)}{4} \quad i = 1, 2.
\]
respectively. Thus, entrepreneur \( i \) determines her effort level \( p_i \) to maximize her expected profit \( \tilde{\pi}^2_{E_i} \), that is,

\[
\max_{p_i} \tilde{\pi}^2_{E_i} = p_i p_j \left( \frac{\Delta}{2} + \frac{\Delta (\theta_1 - \phi)}{6} \right) + p_i (1 - p_j) \left( \frac{\Delta}{2} + \frac{\Delta (2\phi + \theta_1)}{4} \right) - \frac{p_i^2}{2}.
\] (23)

The Nash-equilibrium effort level, \( \tilde{p}^{2*} \), is

\[
\tilde{p}^{2*} = \frac{3\Delta (2(1 + \phi) + \theta_1)}{\Delta (\theta_1 + 8\phi) + 12}.
\] (24)

The corresponding expected profits for the VC and the entrepreneurs are

\[
\tilde{\pi}^2_{V} = \frac{6\Delta^2 (2(1 + \phi) + \theta_1)}{(12 + (\theta_1 + 8\phi) \Delta)^2} (2(1 + \phi)(3 + \Delta\phi) + \theta_1 (\Delta (1 - 2\phi) - 3));
\]

\[
\tilde{\pi}^2_{E_1} = \tilde{\pi}^2_{E_2} = \frac{9 (2(1 + \phi) + \theta_1)^2 \Delta^2}{2 (12 + (\theta_1 + 8\phi) \Delta)^2}.
\]

Note that the ability to switch to a new VC allows the entrepreneurs to extract more surplus from the original VC, with a positive effect on their effort incentives and, thus, the success probability of the start-ups, as summarized in the following lemma.

**Lemma 3** *Entrepreneurial effort is strictly increasing in \( \theta_1 \).*

In addition to ex-post competition, we assume that VCs compete for entrepreneurs ex-ante when they choose their portfolios at \( t = 0 \). Availability of other VCs is important if entrepreneurs and VCs bargain ex-ante for the division of the joint surplus. Ex-ante competition for entrepreneurs provides the entrepreneurs with an outside option which affects the division of the total ex-ante expected surplus.

We further modify the basic game as follows. At the beginning of the game, \( t = 0 \), the VC selects one or two start-ups and bargain with the entrepreneur(s). Bargain results in a side payment, \( t \), from the VC to the entrepreneur(s). We assume that \( t \geq 0 \) since we continue to assume that the entrepreneurs are wealth constrained. Entrepreneurs have now an outside option with a value of \( \theta_0 \geq 0 \). We interpret \( \theta_0 \) as a measure of the scarcity of entrepreneurs relative to VCs, and therefore the tightness of the VC market.\(^{20}\) The VC moves first and chooses whether to form a small or a large portfolio, and therefore, whether to bargain with one or two entrepreneurs. The following proposition characterizes the optimal portfolio size when the entrepreneurs have outside options both ex-ante, \( \theta_0 \), and in the interim, \( \theta_1 \).

\(^{20}\)Note that we assume that at the beginning of the game the VC and the entrepreneurs have already identified each other and that the value for the entrepreneurs to search for alternative VCs is lower than the expected payoff they can obtain from bargaining with the present VC. This is because the search for alternative VCs is costly.

22
**Proposition 8** There are critical values \( \{\hat{\phi}, \hat{\Delta}_1, \hat{\Delta}_2\} \) (defined in the appendix) such that the VC’s optimal portfolio size is as follows:

i) for low project payoff \((0 < \Delta < \hat{\Delta}_1)\) the VC does not invest in any start-up, \( \eta^* = 0 \);

ii) for high project payoff \((\Delta \geq \hat{\Delta}_2)\) the VC invests in one start-up only, \( \eta^* = 1 \);

iii) for moderate project payoff and high focus \((\hat{\Delta}_1 \leq \Delta < \hat{\Delta}_2 \text{ and } \hat{\phi}_c \leq \phi \leq 1)\) the VC invests in two start-ups, \( \eta^* = 2 \).

Furthermore, \( \frac{\partial \Delta^2}{\partial \theta_0} < 0 \).

Proposition 8 shows that our earlier results hold also in the case when entrepreneurs can switch to alternative VCs both ex-ante and in the interim. At the outset of the game, the VC chooses the size of his portfolio by comparing the benefits of large and small portfolios identified in the basic model. When start-ups have a high project payoff, \( \Delta \), the benefits of a small portfolio in terms of better entrepreneurial incentives outweigh the benefits of large portfolio in terms of rent extraction and resource reallocation ability. Thus, the VC optimally chooses a small portfolio and sets \( \eta^* = 1 \). For moderate value of project payoff and high portfolio focus, the VC prefers a large portfolio and sets \( \eta^* = 2 \).

The difference with the basic model is that the entrepreneurs’ outside option due to the availability of other VCs affects the ex-ante allocation of the surplus, and the portfolio size. The ex-ante availability of alternative VCs creates competition for the VC and affects him negatively by reducing his ability to extract surplus from the entrepreneurs. The magnitude of this negative effect is greater (at the margin) in a large portfolio than in a small one. This happens because in a large portfolio the VC must bargain with a greater number of entrepreneurs, each of whom possesses an outside option, \( \theta_0 \). Thus, the availability of other VCs, or an increase in the supply of VCs relative to the supply of entrepreneurs, reduces the desirability of large portfolios. This means that VCs are more likely to choose a smaller portfolio when entrepreneurs have a greater outside option, that is, when entrepreneurs are relatively more scarce than VCs, implied by \( \frac{\partial \Delta^2}{\partial \theta_0} < 0 \). Correspondingly, VCs are more likely to have larger portfolios when there is an increase in the supply of entrepreneurial projects, reflected by a reduction in \( \theta_0 \).

### 5.2 A dynamic extension

In this section we discuss a dynamic, infinite-horizon version of our basic model, where the VC can invest in new start-ups when the existing projects in his portfolio fail or are completed. At \( t = 0 \), the VC can...
invest in either one or two start-ups, or none: \( \eta_0 \in \{0, 1, 2\} \). If the VC invests in one start-up, and the start-up fails at \( t = 1 \), the VC can invest in a new start-up. Investment in a new start-up will require the VC to make a new start-up specific investment. For notational simplicity, we assume that the VC sustains the same cost \( c \) every time he invests in new start-ups.\(^{21}\) If the start-up is successful, the VC and the entrepreneur complete the second stage of the project, as in the static model. The difference from the static model is that now, while bargaining with the entrepreneur, the VC has the ability to invest in a new start-up. This ability allows the VC to extract more surplus from the entrepreneur. Letting \( v_1 \) be the VC’s continuation payoff in the (sub)game, at the bargaining stage the VC’s surplus is given by \( \hat{\ell}_V = v_1 + \frac{(2\Delta \delta - \nu_1)}{2} \). The entrepreneur’s surplus is given by \( \hat{\ell}_{EN} = \frac{(2\Delta \delta - \nu_1)}{2} \), and she chooses her effort level \( p \) to maximize her expected profit \( \hat{\ell}_{EN} \), that is

\[
\max_p \hat{\ell}_{EN} = \frac{p}{2} (2\Delta \delta - v_1) - \frac{p^2}{2},
\]

resulting in \( \hat{p}^{1*} = \delta \Delta - \frac{v_1}{2} \). Stationarity implies that the VC’s expected profit from holding a small portfolio, \( v_1 \), is implicitly defined by

\[
v_1 = \delta \left[ \hat{p}^{1*} (v_1 + \frac{1}{2} (2\Delta \delta - v_1) + \delta v_1) + (1 - \hat{p}^{1*}) v_1 \right] - c,
\]

where \( \delta < 1 \) is the appropriate discount factor.

Now consider the case where the VC invests in two start-ups at \( t = 0 \). If one of the two start-ups fails, state \( SF \), we assume (for simplicity) that it is always optimal for the VC to reallocate his human capital to the successful start-up rather than to invest in an additional one. In this case, the VC and the entrepreneur bargain over the division of the surplus under the condition that the VC has the outside option of investing new start-ups. This implies that the VC will obtain a surplus \( \hat{\ell}_V(SF) = \frac{(1 + \phi) \delta \Delta + v_2}{2} \), leaving \( \hat{\ell}_{EN}(SF) = \frac{(1 + \phi) \delta \Delta - v_2}{2} \) to the entrepreneur. Similarly, if both start-ups are successful, state \( SS \), the VC’s bargaining with both start-ups proceeds as described in the basic model, under the assumption that if bargaining with one start-up breaks down, the VC reallocates his human capital to the remaining start-up and starts a fresh round of bargaining. Bargaining with the remaining start-up takes place again under the condition that the VC has the outside option of investing in a new round of start-ups. This implies that the VC obtains a surplus equal to \( \hat{\ell}_V(SS) = \frac{2(3 + \phi) \delta \Delta}{6} + \frac{v_2}{3} \), leaving \( \hat{\ell}_{EN}(SS) = \frac{3 - \phi \delta \Delta}{6} - \frac{v_2}{6} \).

\(^{21}\)The amount of new start-up specific investment required from the VC will depend, in general, on the transferability of the human capital that the VC has accumulated in the past into new start-ups. This implies that the start-up specific investment may change over time, with later start-ups requiring possibly a lower cost.
to each entrepreneur. Finally, if both start-ups fail, state $FF$, the VC will invest in a new round of two start-ups.

Each entrepreneur chooses her effort level $p_i$ to maximize her expected profit $\hat{\pi}_{E_i}^{2D}$, that is

$$\max_{p_i} \hat{\pi}_{E_i}^{2D} = p_i p_j \left( \frac{3 - \phi}{6} \delta \Delta - \frac{v_2}{6} \right) + p_i (1 - p_j) \left( \frac{1 + \phi}{2} \delta \Delta - v_2 \right) - \frac{p_i^2}{2}. \quad (27)$$

The Nash-equilibrium effort $\hat{p}^{2*}$ is

$$\hat{p}^{2*} = \frac{3 (1 + \phi) \delta \Delta - v_2}{2 (3 + 2 \delta \Delta \phi - v_2)}. \quad (28)$$

Stationarity implies that the VC’s expected profit from holding a large portfolio, $v_2$, is implicitly defined by

$$v_2 = \delta \left[ (\hat{p}^{2*})^2 (\hat{p}_{VC}^{2D}(SS) + \delta v_2) + 2 \hat{p}^{2*}(1 - \hat{p}^{2*})(\hat{p}_{VC}^{2D}(SF) + \delta v_2) + (1 - \hat{p}^{2*})^2 v_2 \right] - c. \quad (29)$$

The following proposition characterizes the VC’s optimal portfolio size.

**Proposition 9** Let $\delta \leq \delta_c$ and $c \leq \hat{c}^D(\delta)$ (defined in the appendix). There are critical values $\{\phi_c^D, \Delta_1^D, \Delta_2^D, \Delta_2^D\}$ such that the VC’s optimal size is as follows:

i) for low project payoff ($0 \leq \Delta < \Delta_1^D$) the VC does not invest in any start-up, $\eta^* = 0$;

ii) for high project payoff ($\Delta \geq \Delta_2^D$) the VC invests in one start-up only, $\eta^* = 1$;

iii) for moderate project payoff and high focus ($\Delta_1^D \leq \Delta < \Delta_2^D$ and $\phi_c^D \leq \phi \leq 1$) the VC invests in two start-ups, $\eta^* = 2$.

Proposition 9 shows that the main results of the static setting extend to the dynamic setting as well. In the dynamic setting, the VC still chooses the size of his portfolio by trading off the rent extraction, the resource allocation, and the value dilution effect. However, the VC’s ability to start new projects affects the magnitude of the rent extraction and the resource reallocation effects. First consider the rent extraction effect. The VC’s ability to start new projects gives the VC an outside option and thus increases his rent extraction ability (both in a small and a large portfolio). However, the marginal increase in his rent extraction ability is greater with a small portfolio than with a large portfolio since in the static model with a small portfolio the VC has no outside options at all. The additional rent extraction ability, however, comes at the cost of worsening entrepreneurial incentives.

Second consider the resource reallocation effect which refers to the VC’s ability to reallocate his human capital and resources from one start-up to another. In the static setting, this effect increases the
attractiveness of a large portfolio relative to a small portfolio since when the VC has only one start-up he cannot reallocate resources. In the dynamic setting, however, the ability to start new investments allows the VC to reallocate resources to new start-ups even when the VC has a small portfolio. Specifically, the VC cannot reallocate his resources across start-ups at a given time, but he can reallocate his resources over time to new start-ups if the current start-up in his portfolio fails or is completed. Hence, the dynamic setting increases the desirability of small portfolios relative to large portfolios, especially for start-ups with moderate fundamentals and high failure rates.

Finally, note that moving from the static to dynamic setting does not affect the value dilution effect in any way, and this effect continues to favor small portfolios in the dynamic setting.

Since all previously identified effects and trade-offs between holding small and large portfolios survive in the dynamic setting, all our previous results go through under the dynamic specification. The only difference is that the dynamic setting may result in an increase or a decrease in the desirability of small portfolios relative to large portfolios depending on parameter values (that is, $\Delta_2^D \gtrless \Delta_2$).

6 Empirical implications

The empirical predictions of our model hinge on the factors that affect the value of the critical parameters in our model, especially $\Delta$ and $\phi$.

(i) VCs prefer to limit their portfolio size for start-ups with ex-ante higher quality. High quality start-ups with strong fundamentals are expected to generate greater value if successful, and therefore are characterized by a greater value of $\Delta$. Our model shows that the VC finds it more desirable to limit his portfolio size for such start-ups because doing so leads to strong entrepreneurial incentives, which proves to be most desirable for start-ups with strong fundamentals. Stronger entrepreneurial incentives improve start-up success probability and, therefore, generate a superior portfolio performance and profits. Thus, this prediction helps explain the finding in Kaplan and Schoar (2005) that better performing VC funds may choose to stay small.

(ii) VCs investing in high-risk technologies manage larger and more focused portfolios. In our analysis, VCs benefit from investing in related start-ups (high $\phi$) because focus allows a more efficient reallocation of human capital from one start-up to another. Reallocation of human capital is more likely when the (endogenous) failure probability for start-ups is relatively high, which happens when $\Delta$ is small or $k$ is large. A greater level of focus reduces the ex-post inefficiency associated with spreading the VC’s resources across
several start-ups, and increases the benefits of ex-post resource reallocation. This implies that focused portfolios are more desirable (all else equal) for start-ups that invest in technologies with high uncertainty and failure rates. This prediction is consistent with the findings of Cumming (2006). This paper documents that large portfolios are more likely to be observed for VCs investing in life sciences (rather than other high-tech firms), and argues that this strategy emerges because there are greater complementarities (i.e., higher focus) across entrepreneurial firms in the life sciences industry.

(iii) VCs expand their portfolios only if they have access to investment opportunities in the sector of their specialization. In our model, forming a portfolio with a high degree of focus benefits the VC in two ways. First, focus allows the VC to extract more surplus from his start-ups and, second, it facilitates the reallocation of human capital and resources among start-ups. In both cases, managing a focused portfolio increases the VC’s investment incentives and rewards his human capital acquisition. This implies that VCs refrain from investing in start-ups that are not related to their area of specialization, and that they are more likely to increase their investments when their industry of specialization experiences a positive technological shock (an increase in investment opportunities). This prediction is consistent with the findings of Gompers, Kovner, Lerner and Scharfstein (2004) which document that venture capital firms with the most industry specific human capital and experience react most to an increase in investment opportunities in the sectors of their specialization. The explanation they offer for this evidence is that it is more difficult for diversified and less specialized VCs to redeploy their human capital from the sectors of their current investment to the sector experiencing an increase in investment opportunities.

(iv) An increase in the availability of VC financing increases entrepreneurial effort and leads to better success rates for start-ups. In Section 5.1 we show that availability of ex-post VC financing increases the entrepreneurs’ ability to extract rents from the VC and, hence, their effort. A greater level of entrepreneurial effort leads to a higher success probability for start-ups. Thus, greater availability of VC financing, and a greater ability of entrepreneurs to transfer their start-ups to alternative VCs, are positively correlated with better success rates for start-ups. This, to our knowledge, a new and testable implication.

(v) In markets with more abundant supply of VCs (relative to entrepreneurs), VCs will hold smaller portfolios. In markets characterized by a more abundant supply of VCs, competition for start-ups allows entrepreneurs to extract greater rents from VCs. As a result, VCs’ desire for larger portfolios decreases. Hence, greater competition among VCs leads to smaller portfolios, a new empirical prediction.

(vi) Industry clustering of smaller entrepreneurial businesses will increase VCs willingness to provide financing to such businesses and will have a positive impact on the creation and the development of new
businesses. Our analysis shows that smaller size entrepreneurial projects characterized by moderate $\Delta$ will become financially viable only if the VC can combine them in his portfolio and create a positive externality between them (given by the reallocation effect). Hence, for a small size start-up, existence of competing but related businesses in the same industry does not necessarily represent a threat, but rather it contributes to the development and commercialization of the business by increasing VCs’ willingness to provide financing for it. This is a novel prediction of our model with the policy implication that encouraging small business development in similar industries or geographical locations will increase prospects of such businesses to receive VC funding.

(vii) VCs with the ability and the experience to create synergies across start-ups will hold larger portfolios. Our model establishes that VCs with a better ability at reallocating resources from one start-up to another are more willing to hold larger portfolios. One interpretation of this result is that experienced VCs will be better at reshuffling resources and generating synergies across start-ups and hence will have a greater willingness to hold larger portfolios.

7 Conclusions

This paper studies the size and focus of a VC’s portfolio. We have identified three main effects of portfolio size on the VC’s and entrepreneurs’ incentives. The first one is the rent extraction effect: The VC can extract higher rents in a larger portfolio by using his ability to reallocate his limited resources from one start-up to another. This effect, everything else constant, leads to stronger incentives for the VC and weaker incentives for the entrepreneurs. The second effect is the resource allocation effect: The VC benefits from investing in a large number of start-ups because this increases (all else equal) the probability that at least one of the start-ups will be successful and thus the VC will have greater chances to earn a return from his investment. This effect depends on the VC’s ability to reallocate ex-post his resources from one start-up to another, after observing whether they have been successful or not. This effect has a positive impact both on the VC’s investment incentives and entrepreneurial incentives. The third effect is the value dilution effect: A larger portfolio requires the VC to spread his limited resources across a large number of start-ups, diluting his value-adding role, with a negative impact on both the VC’s and entrepreneurial incentives.

Our paper has several implications for VC portfolio management. One key message is that limiting portfolio size may prove to be beneficial for a VC despite his ability to add a large number of start-ups
to his portfolio. This result originates from the fact that VC human capital is a scarce resource and committing it to a fewer number of start-ups results in stronger entrepreneurial incentives. In addition, limiting portfolio size is most desirable for start-ups with strong ex-ante fundamentals since improving entrepreneurial incentives is most valuable for such start-ups. A larger portfolio becomes optimal when start-up fundamentals become moderate and when the VC can form a focused portfolio. A high level of focus increases the benefits of a large portfolio in terms of the VC’s rent extraction and resource reallocation ability. We also show that entrepreneurs with smaller businesses may benefit from belonging to a large portfolio, rather than a small one, even if this means that the VC can extract more surplus from them. This happens when a large portfolio is the only way to enable the VC to make the start-up specific investments necessary for the success of the start-ups in his portfolio. In addition, clustering of smaller size businesses with a common industry focus increases their prospects of obtaining VC funding and proves to be beneficial for both entrepreneurs and VCs. Finally, we show that VCs can create value by engaging in portfolio management, a real life practice employed by many VCs. Portfolio management refers to early divestitures of some start-ups to extract higher surplus from the remaining start-ups. We find that the VC benefits from portfolio management when the relatedness of the start-ups in his portfolio is high. Under certain conditions, the ability to engage in portfolio management proves to be socially efficient since it enlarges the set of economically viable start-ups.
References


Appendix

Proof of Proposition 1. The first order condition of (1) with respect to $p$ is $\Delta = kp$, which, if solved for $p$, gives (3). Substituting (3) into the entrepreneur’s and the VC’s objective functions, given by (1) and (2) respectively, gives (4).

Proof of Lemma 1. The VC will make the investment if and only if his expected profits are nonnegative. It is straightforward to see that the VC’s profits, given in (4), are nonnegative if and only if $\Delta \geq \Delta_m = \sqrt{ck}$.

Proof of Proposition 2. Since the reaction functions of the two entrepreneurs are symmetric, the Nash-equilibrium of the effort choice subgame is obtained by setting $p_j \equiv p_i$ in the first-order condition (10), and then solving for $p_i$, giving (11). Substituting the Nash-equilibrium level of effort (11) into the entrepreneurs’ objective function, (8), and in the VC’s objective function, (9), gives the VC’s profits (12) and the entrepreneurs’ profits (13).

Proof of Lemma 2. Direct comparison of $p_1^*$ with $p_2^*$ reveals that $p_1^* > p_2^*$ if and only if

$$\frac{2}{k} > \frac{1 + \phi}{k + \frac{2\Delta^2}{3}},$$

which is always true for $\phi \leq 1$. From (3) and (11) we have that

$$p_1^* - p_2^* = \frac{\Delta}{2k (3k + 2\Delta)} \frac{3k(1 - \phi) + 4\Delta \phi}{2k (3k + 2\Delta) + 2\Delta^2}.$$  (30)

Differentiating (30) with respect to $\Delta$ gives that

$$\frac{\partial (p_1^* - p_2^*)}{\partial \Delta} = \frac{9k^2 (1 - \phi) + 8\Delta \phi (3k + \Delta)}{2k (3k + 2\Delta)^2} > 0.$$

Differentiating (30) with respect to $\phi$ gives that

$$\frac{\partial (p_1^* - p_2^*)}{\partial \phi} = -\Delta \frac{3 (3k - 2\Delta)}{2 (3k + 2\Delta)^2} < 0,$$

since $p_1^*(1) < 1$ implies that $\Delta < k$. Differentiating (30) with respect to $k$ yields that

$$\frac{\partial (p_1^* - p_2^*)}{\partial k} = -\Delta \frac{9k^2 (1 - \phi) + 8\Delta \phi (3k + \Delta)}{2k^2 (3k + 2\Delta)^2} < 0.$$

Proof of Proposition 3. Consider first the case in which the VC selects only one start-up, $\eta = 1$. In this case, from Lemma 1, we have that the VC earns positive expected profits if and only if $\Delta \geq \Delta_m = \sqrt{ck}$.

Consider now the case in which the VC selects a portfolio with two start-ups, $\eta = 2$. Define $\Delta_0(\phi, k)$ implicitly by setting $\pi^2_{V'} = 0$ in (12). It is straightforward to show that $\frac{\partial \pi^2_{V'}}{\partial \Delta_0} > 0$ and $\frac{\partial \pi^2_{V'}}{\partial \phi} > 0$. Thus,
by the implicit function theorem, we have that $\frac{\partial \Delta_1(\phi, k)}{\partial \phi} < 0$. In addition, from $\frac{\partial \pi^{1*}_V}{\partial \Delta} > 0$ it follows that the VC earns positive expected profits only if and only if $\Delta > \Delta_0(\phi, k)$. Part (i) of the proposition is obtained by setting $\Delta_1(\phi, k) \equiv \min \{\Delta_m; \Delta_0(\phi, k)\}$. Note that $\frac{\partial \Delta_1(\phi, k)}{\partial \phi} \leq 0$ since $\frac{\partial \Delta_0(\phi, k)}{\partial \phi} < 0$ and $\Delta_m$ is independent of $\phi$. Define now $\phi_c$ as the unique solution to $\pi^2_{VEC} = 0$ at $\Delta = \Delta_m$, and note that at $\phi = \phi_c$ we have that $\pi^2_{VEC} = \pi^1_{VEC} = 0$. Let now $\phi_c \leq \phi < 1$. In this case, if $\Delta_1 \leq \Delta < \Delta_m$, the VC earns positive expected profits if he invests in two start-ups (since $\Delta > \Delta_1$), but he earns negative expected profits if he invests in one start-up only (since $\Delta < \Delta_m$). Thus, the VC optimally selects two start-ups. Let now $\Delta \geq \Delta_m$. In this case, the VC earns positive expected profits both with one or two start-ups, and both choices are feasible. Therefore, the VC compares his expected profits with one start-up only, $\pi^1_{VEC}$, with his expected profits with two start-ups, $\pi^2_{VEC}$ to decide whether to invest in one or two start-ups. Using (4) and (12), we have that the VC invests in one start-up if and only if

$$\frac{\Delta^2}{k} > \frac{6(3k + \Delta \phi)(1 + \phi)^2 \Delta^2}{4(3k + 2\Delta \phi)^2}. \quad (31)$$

Rearranging the inequality, we obtain that $\pi^1_{VEC} > \pi^2_{VEC}$ if and only if

$$P_1 \equiv 64\phi^2\Delta^2 + 24k\phi(7 - \phi(2 - \phi)) \Delta - 72\phi^2(\phi(2 + \phi) - 1) > 0.$$ 

Note that $P_1$ is a convex parabola in $\Delta$ with two roots, $\hat{\Delta}_1$ and $\hat{\Delta}_2$, given by

$$\hat{\Delta}_1 = \frac{3k}{16\phi} \left( \phi(2 + \phi) - 7 - (1 + \phi) \sqrt{\phi^2 + 2\phi + 17} \right) ,$$

$$\hat{\Delta}_2 = \frac{3k}{16\phi} \left( \phi(2 + \phi) - 7 + (1 + \phi) \sqrt{\phi^2 + 2\phi + 17} \right).$$

It is straightforward to show that $\hat{\Delta}_1 < 0$ for all $0 < \phi < 1$, $k > 0$. We also have that $\hat{\Delta}_2 > 0$ for $\phi > \phi_c$. This can be seen by noting that at $\phi = \phi_c$ we have that $\hat{\Delta}_2(\phi, k) = \Delta_1(\phi, k) = \Delta_m > 0$ and

$$\frac{\partial \hat{\Delta}_2}{\partial \phi} = \frac{3kH(\phi)}{16\phi^2 \sqrt{\phi^2 + 2\phi + 17}} > \frac{3k(7\sqrt{17} - 17)}{16\phi^2 \sqrt{\phi^2 + 2\phi + 17}} > 0,$$

since $H(\phi) \equiv \phi(\phi^2 + \phi - 1) - 17 + (\phi^2 + 7) \sqrt{\phi^2 + 2\phi + 17}$ is increasing in $\phi$. Therefore, $\pi^1_{VEC} > \pi^2_{VEC}$ for $\Delta > \hat{\Delta}_2$ and $\pi^1_{VEC} \leq \pi^2_{VEC}$ for $\Delta_m \leq \Delta < \hat{\Delta}_2$. Define then $\Delta_2(\phi, k) \equiv \hat{\Delta}_2(\phi, k)$ for $\phi_c \leq \phi < 1$. Finally, note that $\Delta_2(\phi, k) = \Delta_1(\phi, k) = \Delta_m > 0$ at $\phi = \phi_c$. Thus, for $0 \leq \phi \leq \phi_c$, $\Delta_m$ implies $\Delta > \hat{\Delta}_2$ since $\frac{\partial \hat{\Delta}_2}{\partial \phi} > 0$ and, hence, that $\pi^1_{VEC} > \pi^2_{VEC}$. Define then $\Delta_2(\phi, k) \equiv \Delta_m$ for $0 \leq \phi < \phi_c$, obtaining (ii) and (iii).

Note that we also have shown that $\frac{\partial \Delta_2(\phi, k)}{\partial k} > 0$ which implies that $\frac{\partial \Delta_m}{\partial k} > 0$. Finally, it is straightforward to see that $\hat{\Delta}_2$ is a linear function of $k$ and positive for $\phi > \phi_c$. This implies that $\frac{\partial \Delta_2}{\partial k} > 0$ for all $\phi > \phi_c$ concluding the proof.
Proof of Proposition 4. By direct calculation, comparing the VC’s payoff from bilateral bargaining $l^B_V(SS)$, given by (14), and that from multilateral bargaining $l^M_V(SS)$, given by (7), reveals that $l^M_V(SS) \geq l^B_V(SS)$ if and only if $\phi \leq \frac{3}{2}$.

Proof of Proposition 5. The first-order condition of (16) is

$$p^B_i (p_j) = (1 + \phi) \frac{4 - 3p_j}{8k} \Delta.$$  \hfill (32)

Since the reaction functions of the two entrepreneurs are symmetric, the Nash-equilibrium of the effort choice subgame is obtained by setting $p_j = p^B_i$ in the first-order condition (32) and solving for $p^B_i$, giving (18). Substituting the Nash-equilibrium level of effort (18) into the entrepreneurs’ objective function (16) and VC’s objective function, (17), gives (19) and (20).

Proof of Proposition 6. The proof of this proposition is similar to the proof of Proposition 3. From (19) define $\Delta^B_i (\phi, k)$ implicitly by setting $\pi^B_{V*} = 0$ in (19). It is straightforward to show that $\frac{\partial \pi^B_{V*}}{\partial \phi} > 0$ and $\frac{\partial \pi^B_{V*}}{\partial k} > 0$. Thus, by the implicit function theorem, we have that $\frac{\partial \Delta^B_i (\phi, k, c)}{\partial \phi} < 0$. Also, from $\frac{\partial \pi^B_{V*}}{\partial \Delta} > 0$, the VC earns positive expected profits only if $\Delta > \Delta^B_1 (\phi, k)$. Define $\Delta$ such that $\pi^B_{V*} = \pi^1_{V*}$ at $\phi = \frac{3}{5}$.

Thus, from (4) there is a $\tilde{c}(k)$ such that we have $\Delta_m < \Delta$ for $c < \tilde{c}(k)$. Since $\frac{\partial \pi^B_i (\phi, k, c)}{\partial \phi} < 0$ it follows that $\Delta^B_1 (\phi, k) < \Delta_m$ for all $\phi_c \leq \phi \leq 1$. Thus, for $0 \leq \Delta < \Delta^B_1 (\phi, k)$, we have that $\pi^B_{V*} < 0$ and $\pi^1_{V*} < 0$, giving part (i) of the proposition. For $\Delta^B_1 (\phi, k) \leq \Delta < \Delta_m$, we have that $\pi^B_{V*} \geq 0$ but $\pi^1_{V*} < 0$. Thus the VC optimally invests in two start-ups, $\eta^* = 2$. For $\Delta \geq \Delta_m$ we have that $\pi^B_{V*} \geq 0$ and $\pi^1_{V*} \geq 0$, and investment in both one or two start-ups is feasible. Therefore, the VC compares his expected profits with one start-up only, $\pi^1_{V*}$, with his expected profits with two start-ups, $\pi^2_{V*}$ to decide whether to invest in one or two start-ups. Using (4) and (19), we have that the VC invests in one start-up if and only if

$$\frac{\Delta^2}{k} > \frac{8 (1 + \phi)^2 \Delta^2 (4k + (1 + \phi) \Delta)}{(8k + 3(1 + \phi) \Delta)^2}.$$  

Rearranging the inequality results in $\pi^1_{V*} > \pi^2_{V*}$ if and only if

$$P_2 \equiv 36 (1 + \phi)^2 \Delta^2 + 8k (1 + \phi) (10 - 2\phi (1 + \phi)) \Delta + 128k^2 (1 - \phi (2 - \phi)) > 0.$$  

Note that $P_2$ is convex in $\Delta$, and has two roots, $\hat{\Delta}^B_1$ and $\hat{\Delta}^B_2$, given by

$$\hat{\Delta}^B_1 = \frac{2k}{9(1 + \phi)} \left(2\phi (2 + \phi) - 10 - \sqrt{A}\right),$$

$$\hat{\Delta}^B_2 = \frac{2k}{9(1 + \phi)} \left(2\phi (2 + \phi) - 10 + \sqrt{A}\right),$$

where $A \equiv (1 + \phi)^2 (4\phi (2 + \phi) + 28)$. It is straightforward to show that $\hat{\Delta}^B_1$ is always negative. $\hat{\Delta}^B_2$ is
positive for $\phi > \hat{\phi}$ since for $c \geq \tilde{c}(k)$ we have that $\hat{\Delta}_2^B > \Delta_m > 0$ for all $1 > \phi > \hat{\phi}$ and
\[
\frac{\partial \hat{\Delta}_2^B}{\partial \phi} = \frac{4k}{18} \frac{(2\phi + \hat{\phi} + 14) \sqrt{A} + 4(1 + \phi)^4}{\sqrt{A}(1 + \phi)^2} > 0.
\]

Hence it follows that $\pi_{1V}^i \geq \pi_{2V}^{B*}$ if $\Delta \geq \hat{\Delta}_2^B$ and $\pi_{1V}^i < \pi_{2V}^{B*}$ if $\Delta_m \leq \Delta < \hat{\Delta}_2^B$. Define then $\Delta_2^B(\phi, k) \equiv \hat{\Delta}_2^B(\phi, k)$, giving part (ii). Finally note that $\frac{\partial \Delta_2^B}{\partial \phi} = \frac{\partial \hat{\Delta}_2^B}{\partial \phi} > 0$, and $\frac{\partial \Delta_2^B}{\partial \phi} \leq 0$.

**Proof of Proposition 7.** By direct calculation, it is possible to show that $\Delta_2^B(\phi, k)$ and $\Delta_2(\phi, k)$ have a unique intersection for $\tilde{\phi} \leq \phi < 1$, which we denote $\tilde{\phi}$. Comparing $\Delta_2^B(\phi, k)$ and $\Delta_2(\phi, k)$ at $\phi = 1$ yields that $\Delta_2^B(\phi = 1, k) = \frac{3}{4} (\sqrt{10} - 1) k \geq \Delta_2(\phi = 1, k) = \frac{3}{4} (\sqrt{5} - 1) k$, which in turn implies that $\Delta_2^B(\phi, k) \leq \Delta_2(\phi, k)$ if and only if $\phi \leq \tilde{\phi}$. From the definitions of $\Delta_2^B(\phi, k)$ and $\Delta_2(\phi, k)$, $\Delta_2^B(\phi, k) = \Delta_2(\phi, k)$ at $\phi = \tilde{\phi}$ implies that $\pi_{2V}^{B*} = \pi_{2V}^i$ at $\phi = \tilde{\phi}$, and which, in turn, implies that $\Delta_2(\phi, k)$ and $\Delta_1(\phi, k)$ intersect only at $\phi = \tilde{\phi}$ for $\tilde{\phi} \leq \phi < 1$, since $\Delta_1^B(\phi, k)$ and $\Delta_1(\phi, k)$ are defined as solutions to $\pi_{2V}^{B*} - c = 0$ and $\pi_{2V}^i(2) - c = 0$, respectively. Calculating $\Delta_2^B(\phi, k)$ and $\Delta_1(\phi, k)$ at $\phi = 1$ yields that $\Delta_1^B(1, k) < \Delta_1(1, k)$, which in turn implies $\Delta_2^B(\phi, k) \geq \Delta_1(\phi, k)$ if and only if $\phi \leq \tilde{\phi}$ given that $\Delta_2^B(\phi, k)$ and $\Delta_1(\phi, k)$ intersect only once for $\tilde{\phi} < \phi < 1$.

**Proof of Lemma 3.** It is immediate to see from $\hat{p}^{1*} = (1 + \theta_1)\Delta$ and from (24) that $\hat{p}^{1*}$ and $\hat{p}^{2*}$ are increasing in $\theta_1$.

**Proof of Proposition 8.** If the VC selects only one start-up, the entrepreneur and the VC bargain over the transfer $t$, where the entrepreneur has the outside option $\theta_0$. Given that the entrepreneur and the VC have the same bargaining power, they will agree on a transfer $t$ such that $\hat{\pi}_{1E}^* + t - \theta_0 = \hat{\pi}_{1V}^* - t$. This implies that $t = \frac{1}{2}(\hat{\pi}_{1V}^* - \hat{\pi}_{1E}^* + \theta_0)$ and that the VC’s payoff, net of the transfer to the entrepreneur is equal to
\[
\hat{\Pi}_{1V}^* = \hat{\pi}_{1V}^* - t = \frac{1}{2}(\hat{\pi}_{1V}^* - \theta_0),
\]
where $\hat{\pi}_{1V}^* \equiv \hat{\pi}_{1V}^* + \hat{\pi}_{1E}^*$ is the joint surplus created by the entrepreneur and the VC.$^{22}$ If the VC selects two start-ups, he will engage in a multilateral bargaining game with the two entrepreneurs similar to the one we described in the basic model. Note that if bargaining with one entrepreneur breaks down, the VC bargain with the remaining one, obtaining $\hat{\Pi}_{1V}^*$. This implies that the VC will agree to pay to entrepreneur $i$ a transfer $t_i$ such that $\hat{\pi}_{2E_i}^* + t_i - \theta_0 = \hat{\pi}_{2V}^* - t_i - t_j - \hat{\pi}_{1V}^*$. Setting $t_i = t_j \equiv t_2$ and $\hat{\pi}_{2E_i}^* = \hat{\pi}_{2E_j}^* = \hat{\pi}_{2V}^*$, we obtain that $t_2 = \frac{1}{3}(\hat{\pi}_{2V}^* - \hat{\pi}_{2E}^* + \theta_0 - \hat{\pi}_{1V}^*)$, and that the VC’s payoff, net of the transfer to the entrepreneur, is equal to
\[
\hat{\Pi}_{2V}^* = \hat{\pi}_{2V}^* - t_2 = \frac{1}{3} \hat{\pi}_{2V}^* + \frac{2}{3} \left[ \hat{\pi}_{2V}^* - \theta_0 - \frac{1}{2}(\hat{\pi}_{1V}^* - \theta_0) \right].
\]

\[22\text{Note that the condition } t \geq 0 \text{ requires that } \hat{\pi}_{1V}^* - \hat{\pi}_{1E}^* \leq \theta_0, \text{ which we assume to be the case.} \]
Let $\tilde{\pi}^2* = \tilde{\pi}_V^2 + 2\tilde{\pi}_E^2$ be the joint surplus created by the VC and the two entrepreneurs. This proof now follows a procedure similar to the proof of Proposition 3. Consider first the case in which the VC selects only one start-up, $\eta = 1$. In this case, from (33), we have that $\tilde{\Pi}_V^1* \geq 0$ for $\Delta \geq \tilde{\Delta}_m \equiv \sqrt{\frac{2c}{\pi + \theta_0(2\phi-\theta_0)}}$. Consider next the case in which the VC selects two start-ups, $\eta = 2$. Define $\tilde{\Delta}_0(\phi)$ implicitly by setting $\tilde{\Pi}_V^2* = 0$ in (34). By direct calculation it is easy to verify that $\frac{\partial \tilde{\Delta}_0(\phi)}{\partial \phi} > 0$ and $\frac{\partial \tilde{\Pi}_V^2*}{\partial \phi} > 0$. Thus, by the implicit function theorem, we have that $\frac{\partial \tilde{\Delta}_0(\phi)}{\partial \Delta} < 0$. In addition, from $\frac{\partial \tilde{\Pi}_V^2*}{\partial \Delta} > 0$ it follows that the VC earns positive expected profits for $\Delta > \tilde{\Delta}_0(\phi)$. Part (i) of the proposition is obtained by setting $\tilde{\Delta}_1(\phi) = \min\{\Delta_m; \tilde{\Delta}_0(\phi)\}$. Note that $\frac{\partial \tilde{\Delta}_1(\phi)}{\partial \phi} \leq 0$ since $\frac{\partial \tilde{\Delta}_0(\phi)}{\partial \phi} < 0$ and $\Delta_m$ is independent of $\phi$. Define now $\phi_c$ as the unique solution to $\tilde{\Pi}_V^2* = 0$ at $\Delta = \tilde{\Delta}_m$, and note that at $\phi = \phi_c$ we have that $\tilde{\Pi}_V^2* = \tilde{\Pi}_V^1* = 0$. Let now $\phi_c \leq \phi \leq 1$. In this case, if $\tilde{\Delta}_1 \leq \Delta < \tilde{\Delta}_m$, the VC earns positive profits if he invests in two start-ups (since $\Delta \geq \tilde{\Delta}_1$), but he earns negative profits if he invests in one start-up only (since $\Delta < \tilde{\Delta}_m$). Thus, the VC optimally selects two start-ups. Let now $\Delta \geq \tilde{\Delta}_m$. In this case, the VC earns positive expected profits both with one or two start-ups, and both choices are feasible. Therefore, the VC compares his expected profits with one start-up only, $\tilde{\Pi}_V^1*$, with his expected profits with two start-ups, $\tilde{\Pi}_V^2*$. From (33) and (34) we obtain that $\tilde{\Pi}_V^1* \geq \tilde{\Pi}_V^2*$ if and only if

$$\frac{1}{2} \tilde{\pi}^1* - \tilde{\pi}^2* \geq -\frac{3\theta_0}{2},$$

that is, if and only if

$$\Delta^2 F(\Delta, \phi) \geq -\frac{3\theta_0 + c}{2},$$

where $F(\Delta, \phi) \equiv \frac{3 + \theta_1(1 - \phi) - 3(2(1 + \phi) + \theta_1)}{\pi + \theta_0(2\phi - \theta_0)}$. By direct differentiation, it can be easily verified that $\frac{\partial F(\Delta, \phi)}{\partial \Delta} > 0$ and $\frac{\partial F(\Delta, \phi)}{\partial \phi} < 0$. This implies that there is a $\tilde{\Delta}_2(\phi, \theta_0)$ such that $\tilde{\Pi}_V^1* \geq \tilde{\Pi}_V^2*$ if and only if $\Delta \geq \tilde{\Delta}_2(\phi, \theta_0)$, with $\frac{\partial \Delta(\phi, \theta_0)}{\partial \phi} > 0$ implied by implicit function differentiation. This implies that for $\tilde{\Delta}_1 \leq \Delta < \tilde{\Delta}_2(\phi, \theta_0)$ the VC sets $\eta* = 2$, and for $\Delta \geq \tilde{\Delta}_2(\phi, \theta_0)$ the VC sets $\eta* = 1$, proving (iii) and (iv). This also implies that $\frac{\partial \Delta(\phi, \theta_0)}{\partial \phi} < 0$. Part (ii) of the proposition is easily verified by noting that $\tilde{\Delta}_2(\phi_c) = \tilde{\Delta}_1(\phi_c) = \Delta_m > 0$ at $\phi = \phi_c$. Thus, $\frac{\partial \Delta(\phi, \theta_0)}{\partial \phi} > 0$ implies that for $0 \leq \phi < \phi_c$ and $\Delta \geq \Delta_m$, we have that $\Delta > \tilde{\Delta}_2$ and, hence, that $\tilde{\Pi}_V^1* > \tilde{\Pi}_V^2*$, concluding the proof.

**Proof of Proposition 9.** This proof follows a procedure similar to the one used in the proof of Proposition 3. The VC will choose at each date the portfolio that maximizes his continuation payoff by comparing (26) and (29). By implicit function differentiation of (29) it is easy to show that there is a $\delta_c > 0$ such that if $\delta \leq \delta_c$ we have that

$$\frac{\partial \Pi^2}{\partial \phi} > 0 \quad \text{and} \quad \frac{\partial \Pi^2}{\partial \Delta} > 0.$$  

(35)
This implies that for all \( \Delta \) there is a unique \( \phi^D(\Delta) \in [0, 1] \) such that \( v^2(\phi, \Delta) = v^1(\Delta) \phi^D(\Delta) \). Define \( \Delta^D_m \) such that \( v^1(\Delta^D_m) = 0 \). Let \( \Delta^D_\delta(\phi) \) be such that \( v^2(\phi, \Delta^D_\delta(\phi)) = 0 \), and let \( \phi^\Delta \) such that \( \Delta^D_\delta(\phi^\Delta) = \Delta^D_m \). Note that \( v^1(\Delta^D_m) = v^2(\phi^\Delta, \Delta^D_m) = 0 \). By the implicit function theorem, (35) implies that \( \frac{\partial \Delta^D_\delta(\phi)}{\partial \phi} < 0 \).

Define then \( \Delta^D_\phi(\phi) \equiv \min\{\Delta^D_m; \Delta^D_\delta(\phi)\} \). Monotonicity of \( v^2(\phi, \Delta) \) and \( v^1(\Delta) \) in \( \Delta \) imply that \( v^2(\phi, \Delta^D) < 0 \) and \( v^1(\Delta) < 0 \) for \( \Delta < \Delta^D_\phi(\phi) \), and therefore that the VC will not invest in any start-up for \( \Delta < \Delta^D_\phi(\phi) \), that is, \( \eta^* = 0 \). For \( \Delta \geq \Delta^D_\phi(\phi) \) the VC will invest in either one or two start-ups. Let \( \Delta^D(\phi) \) be the inverse of \( \phi^D(\Delta) \). If, for some \( \phi \), \( \Delta^D(\phi) \) is not single-valued, let \( \Delta_2(\phi) = \min\{\Delta : \phi^D(\Delta) = \phi\} \) and \( \Delta_2(\phi) = \max\{\Delta : \phi^D(\Delta) = \phi\} \). Let now \( \Delta^{Max} \equiv \frac{c}{2(1-\delta(1+\delta))} \). From \( \hat{p}^{1*} = \delta \Delta - \frac{v_1}{2} \), it is easy to verify that \( \hat{p}^{1*}(\Delta^{Max}) = 1 \), which means that \( \Delta \leq \Delta^{Max} \). Consider now \( \hat{p}^{2*}(\phi, \Delta, \delta, v_2) \) as defined in (28). By direct differentiation, it is easy to verify that \( \frac{\partial \hat{p}^{2*}(\phi)}{\partial \phi} > 0 \) at \( \delta = 0 \). This implies that there is a \( \delta_{c_2} > 0 \) such that \( \frac{\partial \hat{p}^{2*}(\phi)}{\partial \phi} > 0 \) for all \( \delta \in [0; \delta_{c_2}] \). Furthermore, by direct differentiation, it is also easy to verify that \( \frac{\partial \hat{p}^{2*}(\phi)}{\partial v_2} < 0 \) and \( \frac{\partial \hat{p}^{2*}(\phi)}{\partial \Delta} > 0 \). This implies that

\[
\hat{p}^{2*} = \frac{3}{2} \left( 1 + \frac{\delta \Delta - v_2}{2} \right) < \frac{3}{2} \left( \frac{2 \delta \Delta}{3 + 2 \delta \Delta} \right) < \psi(\delta, c) = \frac{3}{2} \frac{2 \delta \Delta^{Max}}{3 + 2 \delta \Delta^{Max}}.
\]

Since \( \psi(\delta, 0) = 0 \), there is a \( c^D(\delta) \) such that \( \hat{p}^{2*} < \frac{1}{2} \) for all \( c \in [0, c^D(\delta)] \). By implicit function differentiation of (29) it is easy to verify that in this case \( \frac{\partial \hat{p}^{2*}}{\partial \phi} > 0 \). Thus, by substituting \( \hat{p}^{2*} = \frac{1}{2} \) in (29) we obtain that \( v_2 \leq \bar{v}_2 \), where

\[
\bar{v}_2 = \delta \left[ \frac{1}{4} \left( 4 \frac{\delta \Delta^{Max} + \bar{v}_2}{3} + \frac{2 \Delta^{Max} + \bar{v}_2}{4} \right) + \frac{1}{4} v_2 \right] - c.
\]

Thus, solving for \( \bar{v}_2 \), we obtain

\[
v_2(\Delta^{Max}, \delta) \leq \bar{v}_2 = \frac{\delta c}{\delta c + 4 \delta (1-\delta(1+\delta)) - c}.
\]

Similarly, from (26), we obtain that

\[
v^1(\Delta^{Max}, \delta) = \frac{c}{2 \delta (1 - \delta(1 + \delta))}.
\]

Comparing \( \bar{v}_2 \) with \( v^1(\Delta^{Max}) \) we obtain that \( v_2(\Delta^{Max}, \delta) < v^1(\Delta^{Max}, 0) \). Thus, there is a there is a \( \delta_{c_3} > 0 \) such that \( v_2(\Delta^{Max}, \delta) < v^1(\Delta^{Max}, \delta) \) for all \( \delta \in [0; \delta_{c_3}] \). Define \( \delta_{c_3} \equiv \min\{\delta_{c_1}; \delta_{c_2}; \delta_{c_3}\} \). This implies that \( v_1(\phi, \Delta) \geq v_2(\Delta^{Max}, \delta) \) for \( \Delta \geq \Delta^D_\phi(\phi) \), and that \( v_1(\phi, \Delta) < v_2(\Delta) \) for \( \Delta^D_\phi(\phi) \leq \Delta < \Delta^{Max} \), where \( \Delta^D_\phi(\phi) = \Delta_2(\phi) \), concluding the proof.